HARMONIC OSCILLATIONS

MECHANICAL AND ELECTRICAL OSCILLATORS:

- Mechanical oscillator is defined as the mechanical energy of the oscillator is spent in overcoming "air- resistance" . The oscillations die out with time.
  Ex: Simple pendulum, Loaded – spring system.
- Electrical oscillator is defined as the oscillator in which electrical energy is slowly dissipated in the form of heat in the resistance so the resistance causes the damping. Thus the electrical oscillator is damped.
  Ex: Series and Parallel LCR circuit.

S.NO	MECHANICAL OSCILLATOR	ELECTRICAL OSCILLATOR
1	Mass of the mechanical oscillator.	Corresponds to inductance of the electrical oscillator.
2	Fractional force per unit velocity.	Corresponds to resistance(R).
3	Force constant.	Corresponds to 1/C
4	Displacement(y)	Corresponds to charge(q).
5	Velocity (dy/dt).	Corresponds to current(dq/dt).

DAMPED MOTION/VIBRATIONS/OSCILLATIONS:

For an ideal simple harmonic oscillator executing SHM the amplitude of oscillator remains constant for an infinite medium in absence of frictional forces and other pseudo forces etc.

Ex; Motion of clock or simple pendulum in vacuum.

i.e. we say that for an ideal SHO the oscillations or motion is un damped. Whereas when a body vibrates in air or other material medium the amplitude of vibration does not remains constant but gradually decreases and finally the body comes to rest on account of frictional forces either inside material of the body or outside, in the medium such as air. Now the body is said to be execute ***damped vibrations***.

Definition: Damped harmonic motion is defined as decay or decrease of amplitude of motion with respect to time in presence of air or other medium, it is "damped simple harmonic motion". The oscillator which performs such type of motion is known an "Damped harmonic oscillator".

Ex: Motion of tuning fork, Motion of simple pendulum. Etc.

Differential equation for damped simple harmonic oscillator:

Consider a particle executing damped SHM about the equilibrium position. Let 'y' be the displacement from the equilibrium position at any instant 't' and (dy/dt) its velocity at that instant. The two forces that are acting on the particle at the instant 't' are

(i) A restoring force proportional to the displacement, but oppositely directed, given by

$$F = -sy$$

Wher 's' is the force per unit displacement (force constant).

(ii) A frictional force proportional to the velocity, but oppositely directed given by

$$F_R = -r\,(dy/dt)$$

Where 'r' is the frictional force per unit velocity.

Therefore, The total force acting on the particle at the instant 't' is given by

$$F = -s\,y - r\,(dy/dt) \qquad (1)$$

If ‘m’ be the mass of the particle and d^2y / dt^2 the acceleration at the instant ‘t’ , then by Newton’s second law of motion we write

$$F = m\,(d^2y / dt^2) \qquad (2)$$

From eq.(1) & (2)

$$m\,(d^2y / dt^2) = -\,s\,y - r\,(dy/dt)$$

$d^2y / dt^2 + (r/m)(dy/dt) + (s/m)\,y = 0$

Let $r/m = 2K$ & $s/m = \omega^2$

We get, $d^2y / dt^2 + 2K\,(dy/dt) + \omega^2 y = 0$ (3)

Eq. (3) represents the differential equation of damped harmonic oscillator.

The solution for above eq. is

$Y = C\,e^{\alpha t}$ (4)

Where C & $\propto$ are arbitrary constants.

Differentiating eq. (4) w.r.t ‘t’ we get

$dy/dt = C\propto e^{\alpha t}$ & $d^2y / dt^2 = C\propto^2 e^{\alpha t}$

substituting these values in eq. (3), we get

$C\propto^2 e^{\alpha t} + 2KC\propto e^{\alpha t} + \omega^2 C\,e^{\alpha t} = 0$

$\propto^2 + 2K\propto + \omega^2 = 0$

$\propto = (-2K \pm \sqrt{4K^2 - 4\omega^2})/2$

$\propto = -K \pm \sqrt{K^2 - \omega^2}$ (5)

Since, there are two values of ‘$\propto$’ and the general solution of eq. (3) is

$Y = A_1\, e^{(-K+\sqrt{K^2-\omega^2})t} + A_2\, e^{(-K-\sqrt{K^2-\omega^2})t}$ (6)

Where A_1 & A_2 are constants. The values of these constants can be determined from the boundary conditions.

Types of Damping:

Depending on the relative values of “ K ” & “ω” three cases are possible.

<u>Case – I</u> <u>Over damped/ Heavy damped motion :</u>

When $K^2 > \omega^2$, In this case $\sqrt{K^2 - \omega^2}$ is real & less than ‘K’. Now the powers in eq. (6) are both (-ve), this displacement ‘y’ consists of two terms, both dying to zero without performing any oscillations as shown in fig. The rate of decrease of displacement is governed by the term as the other term reduced to zero quickly relative to it.

Therefore, In case of over damped motion, the body once displaced return to its equilibrium position quite slowly without performing any oscillation. It is also known as over damped of dead beat.

Ex: Motion of simple pendulum moving in a thick oil or by a dead beat moving coil galvanometer.

<u>Case – II</u> <u>Critical damping :</u>

If we put, $K^2 = \omega^2$, in equation (6) then, this solution does not satisfy the differential equation. Let us consider $\sqrt{K^2 - \omega^2}$ is not zero, but this is equal to a very small quantity ‘h’. i.e. $\sqrt{K^2 - \omega^2} = h$

Now eq. (6) reduced to

$Y = A_1\, e^{(-K+h)t} + A_2\, e^{(-K-h)t}$

$Y = e^{-kt}\,(A_1\, e^{(-K+h)t} + A_2\, e^{(-K-h)t})$

$Y = e^{-kt}\,[A_1\,(1+ht+......) + A_2\,(1-ht+......)]$

$Y = e^{-kt}\,[(A_1 + A_2) + (A_1 - A_2)\,ht]$

$Y = e^{-kt}\,[p + q\,t]$ (6)

Where $p = A_1 + A_2$ & $q = (A_1 - A_2)\,h$

In this equation as ‘t’ increases, e^{-kt} decreases and [p + q t] increases and the damping is said to be critical. Thus in the case of critical damping, the motion just becomes a periodic or non oscillatory. This type of motion is exhibited by many pointer instruments such as “ Ammeter, voltmeter, etc”, in which the pointer moves to the correct position and comes to rest much rapidly without any oscillations.

Case – III Under damped motion :

When $K^2 < \omega^2$, In this case $\sqrt{K^2 - \omega^2}$ is imaginary.

$\sqrt{K^2 - \omega^2} = \sqrt{-1}\sqrt{\omega^2 - K^2} = i\beta$

Where, $\sqrt{\omega^2 - K^2} = \beta$ and $\sqrt{-1} = i$

Therefore, eq. (6) now becomes

$Y = A_1\, e^{(-K+i\beta)t} + A_2\, e^{(-K-i\beta)t}$

$Y = e^{-kt}\,[A_1\, e^{i\beta t} + A_2\, e^{-i\beta t}\,]$

$Y = e^{-kt}\,[A_1\,(\cos\beta t + i\sin\beta t\,) + A_2\,(\cos\beta t - i\sin\beta t)\,]$

$Y = e^{-kt}\,[\,(A_1+A_2)\cos\beta t + I\,(A_1 - A_2)\ \sin\beta t]$

Where $A_1+A_2 = a\sin\emptyset$ and $A_1 - A_2 = a\cos\emptyset$

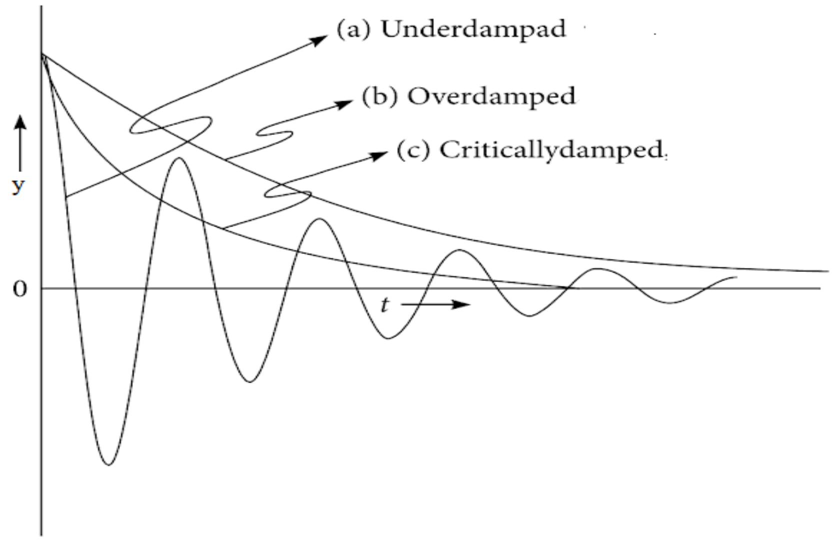
$Y = a\,e^{-kt}\ \sin(\beta t + \emptyset)$

$Y = a\,e^{-kt}\ \sin(\sqrt{\omega^2 - K^2}\,t + \emptyset)$

The above equation represents the SHM with amplitude a e^{-kt} and time period is equal to $T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2 - K^2}}$

The amplitude of the motion is continuously decreasing to factor e^{-kt} which is called the damping factor. Because of the value of $\sin(\sqrt{\omega^2 - K^2}\,t + \emptyset)$ varies between ae^{-kt} and -ae^{-kt} . The decay of the amplitude depends upon the damping factor ‘K’. It is called “under damped motion”. In this case period is slightly increased or frequency decreased because the period is now $\frac{2\pi}{\sqrt{\omega^2 - K^2}}$ while in the absence of damping it was $T = \frac{2\pi}{\omega}$

Ex: Motion of a pendulum in air, The electric oscillations of LCR circuit.



Return of a harmonic oscillator to equilibrium position. (a) Under damped (b) Over damped (c) Critically damped

ENERGY OF DAMPED HARMONIC OSCILLATOR:

Whenever a system vibrates, it is subjected to frictional force due to air resistance or forces within the system itself. These forces opposes the motion of the system. The work done against the forces is dissipated out of the system as heat. So the total mechanical energy of the vibrating system decreases with time. Hence, the system vibrates with gradually decreasing amplitude and finally comes to rest. Now it is required to obtain an expression for the power dissipated from the harmonic oscillator.

The displacement 'Y ' of a damped harmonic oscillator at any instant is given by

$Y = a\, e^{-kt} \sin(\omega' t + \emptyset)$ (1)

Where, a & $\emptyset$ are arbitrary constants, 'K' is damping constants and $\boldsymbol{\omega'} = \sqrt{\boldsymbol{\omega^2 - K^2}}$ is the angular frequency of the damped oscillator

$\boldsymbol{\omega} = \sqrt{\boldsymbol{s/m}}$ where $\boldsymbol{\omega}$ is the angular frequency of the un damped oscillator and 'S' is the force constant.

The instant velocity 'V' is given by

$V = dy/dt = -Ka\, e^{-kt} \sin(\omega' t + \emptyset) + a\, e^{-kt} \cos(\omega' t + \emptyset)\, \omega'$

$V = a\, e^{-kt} [-K \sin(\omega' t + \emptyset) + \cos(\omega' t + \emptyset)\, \omega']$

In practice, the damping is very small, so that $K << \boldsymbol{\omega}$ so the term $- K \sin(\omega' t + \emptyset)$ can be neglected. Hence, the above equation can be written as

$V = a\, e^{-kt} \cos(\omega' t + \emptyset)\, \omega'$ (2)

Thus the total mechanical energy 'E' of the oscillator may be written as

$E = K.E + P.E$

$E = ½\, m v^2 + 1/2\, SY^2$

Substituting the values of 'Y' and 'V' from eq. (1) and (2) we get

$E = ½\, m a^2 e^{-2kt} \omega'^2 \cos^2(\omega' t + \emptyset) + 1/2\, s a^2 e^{-2kt} \sin^2(\omega' t + \emptyset)$

Further $K << \boldsymbol{\omega}$, we can write $\omega'^2 = \boldsymbol{\omega^2 - K^2} = \boldsymbol{\omega^2} = \boldsymbol{s/m}$

$E = ½\, m a^2 e^{-2kt} (s/m) \cos^2(\omega' t + \emptyset) + 1/2\, s a^2 e^{-2kt} \sin^2(\omega' t + \emptyset)$

$E = ½\, s a^2 e^{-2kt} [\cos^2(\omega' t + \emptyset) + \sin^2(\omega' t + \emptyset)]$

$E = ½\, s a^2 e^{-2kt}$ (3)

Above eq. shows that the energy of the oscillator decreases with time. But rate at which energy lost is the power dissipation.

$P = -dE/dt = -[½\, s a^2 (-2k)\, e^{-2kt}]$

$P = 2KE$ (4)

QUALITY FACTOR (Q):

It is defined as 2π times the ratio of the energy stored in the system to the energy lost per period.

$$Q = 2\pi \frac{\text{Energy stored in system}}{\text{Energy lost per period}}$$

$$Q = 2\pi \frac{E}{PT}$$

Where, P is the power dissipation and T is the time period.

FORCED DAMPED HARMOINC OSCILLATOR:

FORCED VIBRATIONS: Forced vibrations can be defined as the vibrations in which the body vibrates with a frequency other than it's natural frequency under the action of an external periodic force. The oscillator which executes forced harmonic vibrations is known as Forced Harmonic Oscillator.

Equation of motion of forced damped harmonic oscillator:

The forces acted upon the particle is

i. A restoring force proportional to the displacement but oppositely directed given by $-ky$

ii. A frictional force proportional to velocity but oppositely directed given by –r dy/dt.

Where 'r' is frictional force per unit velocity

iii. The external force represented by **F cos ωt.** **(**where 'ω' is angular frequency**).**

Where 'F' is the maximum value of this force

The total force acting on the particle is given by

$$F = -ky - r\, dx/dt + F\cos\omega t$$

$$m\,(d^2y/dt^2) = -ky - r\, dy/dt + F\cos\omega t$$

$$m\,(d^2y/dt^2) + ky + r\, dy/dt = F\cos\omega t$$

divided by above equation by 'm'

$$(d^2y/dt^2) + (k/m)y + (r/m)\, dy/dt = (F/m)\cos\omega t \qquad (1)$$

Let r/m = 2b, $\frac{k}{m} = \omega_0^2$ and F/m = f

$$d^2y/dt^2 + 2b\, dy/dt + \omega_0^2 y + = f\cos\omega t \qquad (2)$$

The above eq. is the equation of motion of the forced oscillator.

The forced vibrator oscillates with the frequency of periodic force ω instead of its natural frequency ω_0 .

The solution for equation (2) is

$$y = A\cos(\omega t - \theta) \qquad (3)$$

where, A is the amplitude and θ is the phase difference between displacement of oscillator and external periodic force. A, θ are the constants to be determined.

Differentiating eq. (3) twice with respect to 't'

$$dy/dt = -A\omega\sin(\omega t - \theta)$$

$$d^2y/dt^2 = -A\omega^2\cos(\omega t - \theta) \qquad (4)$$

substituting the eq. (3) and (4) in eq. (2)

$$-A\omega^2\cos(\omega t - \theta) - 2bA\omega\sin(\omega t - \theta) + \omega_0^2 A\cos(\omega t - \theta) = f\cos\omega t$$

$$A(\omega_0^2 - \omega^2)\cos(\omega t - \theta) - 2bA\omega\sin(\omega t - \theta) = f\cos[(\omega t - \theta) + \theta]$$

$$A(\omega_0^2 - \omega^2)\cos(\omega t - \theta) - 2bA\omega\sin(\omega t - \theta) = f\cos(\omega t - \theta)\cos\theta - f\sin(\omega t - \theta)\sin\theta \qquad (5)$$

Equate the coefficients of cos(ωt - θ) and sin(ωt - θ) on both sides,

$$\therefore A(\omega_0^2 - \omega^2) = f\cos\theta \qquad (6)$$

$$2bA\omega = f\sin\theta \qquad (7)$$

Squaring and adding above two equations

$$A^2\left(\omega_0^2 - \omega^2\right)^2 + 4b^2A^2\omega^2 = f^2$$

$$A = \frac{f}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\,b^2\omega^2}} \qquad (8)$$

Dividing eq. (7) by eq. (6)

$\tan\theta = \frac{2b\ \omega}{\left(\omega_0{}^2 - \omega^2\right)}$

$\theta = \tan^{-1}\left[\frac{2b\ \omega}{\left(\omega_0{}^2 - \omega^2\right)}\right]$ (9)

Substituting the equations (8) and (9) in equation (3)

$y = \frac{f}{\sqrt{\left(\omega_0{}^2 - \omega^2\right)^2 + 4\ b^2\omega^2}} \cos(\omega t - \theta)$ (10)

The equations (8) and (9) gives the amplitude and phase of forced vibrations.

UNIT II WAVE OPTICS

Introduction: Optics is a branch of physics which deals with the “Theory of light and its propagation in a given medium”. The branch of optics is divided into two parts

1. Ray or Geometrical optics and
2. Physical or Wave optics

Ray optics: Ray optics deals with image formation by optical systems. It was supported by Newton’s corpuscular theory. It deals with the particles propagating in a medium called “light-photons with energy $E_n = nh\upsilon$ in a medium”.

Ray optics deals with reflection, refraction, Snell’s law,total internal reflection,image formation in lens and mirrors, etc.

Physical optics: Physical optics deals with nature of light. Huygens proposed the wave theory of light. According to this, a luminous body is a source of disturbance in a hypothetical medium called “ether”

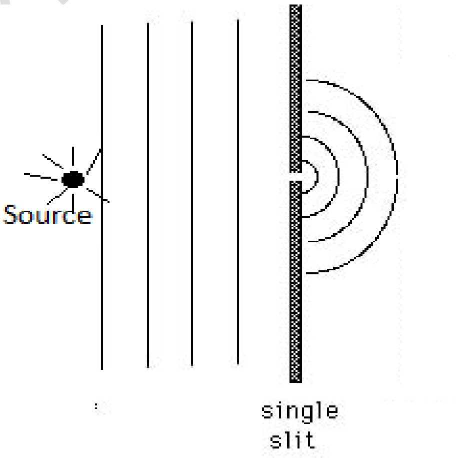
Huygen’s principle:

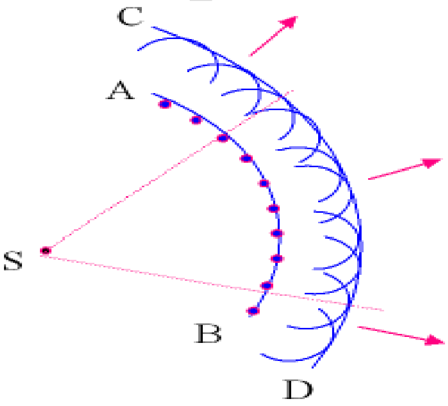
Huygens principle provides a geometrical method of finding the shape and position of the wave front at a certain instant from its shape and position of some earlier instant.

Huygen’s principle is stated in the following two parts.

i. Each point on the wavefront acts as a centre of new disturbance and emits its own set of spherical waves called secondary wavelets. These secondary wavelets travel in all directions with the velocity of light so long as they move in the same medium.

ii. The envelope or the locus of these wavelets in the forward direction gives the position of the new wavefront at any subsequent times.





Huygens' Principle:
Each wavefront is the envelope of the wavelets. Each point on a wavefront acts as an independent source to generate wavelets for the next wavefront. AB and CD are two wavefronts.

Superposition Principle: When two or more waves arrive at a point in space simultaneously, the net wave disturbance at that point and at any given time is the vector sum of all the wave disturbances at that point at that particular time. This is called the principle of superposition(“The resultant or total displacement of the medium acted upon by two or more waves simultaneously equal to sum or difference of displacements of individual waves”.)

$$R \text{ or } Y = y_1 \pm y_2 \pm ---------- \pm y_n$$

Interference of light:

It is defined as “Modification of Resultant Intensity of light obtained by the superposition of two or more light waves”. This theory of interference of light was developed by Thomas young in his experimental study of light.

The resultant intensity consists of series of bright & dark fringes that appear on the screen which are known as interference pattern or interference fringes which correspond to maximum & minimum intensities of light.

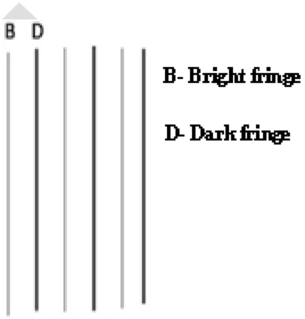


Fig: Interference fringe pattern

Interference pattern may be either straight or circular or parabolic (Arc) fringes

Coherent Sources: If the phase difference between two light waves emitted from two sources is zero or has a constant value then the sources are said to be coherent.

Incoherent- sources: If the phase difference between two light waves coming from two sources changes with time, the sources are called as “In-coherent Sources”.

Types of interference: Interference of light based upon young’s double slit experiment is divided into two types.

1. Constructive interference
2. Destructive interference

Constructive interference: When the two light waves reach a point in phase the resultant displacement (R_{Max} or Y_{Max}) is always equal to algebraic sum of individual displacements of the light waves. It is known as constructive interference.

$$R_{Max} \text{ or } Y_{Max} = y_1 + y_2 + \_\_\_\_\_\_ + y_n$$

Destructive interference: When the two light waves reach a point i.e. out of phase. The resultant displacement (R_{Min} or Y_{Min}) is always equal to the difference of displacements of the light waves. It is known as destructive interference.

$$R_{Min} \text{ or } Y_{Min} = y_1 - y_2 \_\_\_\_\_\_\_\_ - y_n$$

Conditions for interference of light:

1) The two light sources emitting light waves should be coherent.
2) The separation between the two sources should be small.
3) The distance between the two sources and the screen should be large.
4) To view interference fringes, the back-ground (good-contrast) should be dark.
5) Amplitudes of two light waves are nearly equal.
6) The source should be mono-chromatic.
7) The sources should be narrow i.e. they must be small.

Two produce coherent beam of light, two techniques are adopted. They are

1. Interference by division of wave front.
2. Interference by division of amplitude.

Interference by division of wave front: In this, the coherent sources are obtained by dividing incident wave front in to two parts by utilizing reflection, refraction or diffraction. Here point source should be used.
Examples: Fresnel's bi prism, Fresnel's mirrors, Lloyd's mirror, lasers etc.

Interference by division of amplitude: In this, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Here point or broad source can be used.
Examples: the interference in thin films, newton's rings, Michelson's interferometer.

Newton's Rings:

Newton's rings are one of the best examples for the interference in a non-uniform thin film. When a plano convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero.

If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are seen. These circular rings were discovered by Newton and are called as Newton's rings.

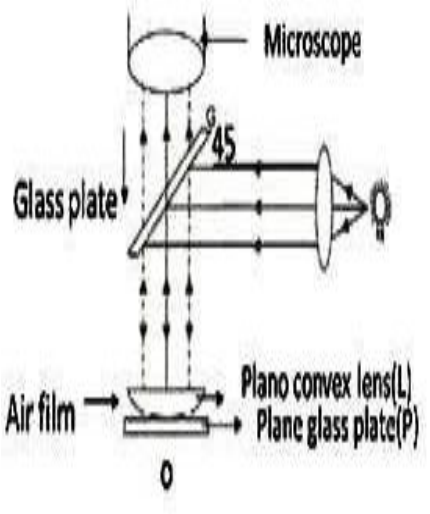


Fig 2 Newton's rings experiment

The plano-convex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate (P). The lens makes the contact with the plate at 'O'. The monochromatic light falls on a glass plate G held at an angle of 45^0 with the vertical. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P. A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen near the upper surface of the air film through the microscope.

Explanation:

Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens. A part of the incident light is reflected at a point in the form of the ray (1) with any additional phase change. The other part is refracted and again reflected in the form of the ray (2) with additional phase change of π or path change of λ/2.

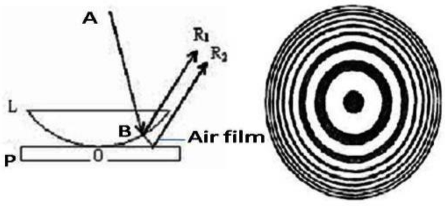


Fig: formation of Newton's rings

As the rings are observed in the reflected light, the path difference between the rays is

$$pathdiffernce = 2\mu t\cos r + \frac{\lambda}{2}$$

For air film μ = 1 and for normal incidence r = 0,

then, $$pathdifference = 2t + \frac{\lambda}{2}$$

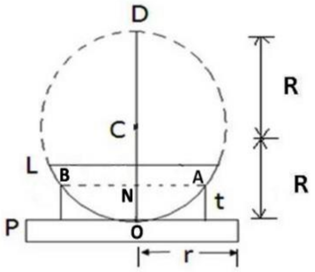
At the point of contact t =0 the path difference is λ/2, i.e., the reflected light at the point of contact suffers phase change of π. Then the incident and reflected rays are out of phase and interface destructively. Hence the central spot is dark.

The condition for the bright ring is $2t + \frac{\lambda}{2} = n\lambda$

$2t = (2n-1)\frac{\lambda}{2}$ where n=1, 2, 3…..

The condition for dark ring is $2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$2t = n\lambda$ where n=0, 1, 2, 3……



Theory: To find the diameters of dark and bright rings, let 'L' be the lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness't'.

From the property of a circle,

$$NA \times NB = NO \times ND$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

where R is the radius of curvature of plano lens and‘t’ is the maximum thickness of air film

As ‘t’ is small, r^2 will be negligible, $r^2 = 2Rt$

$$t = \frac{r^2}{2R}$$

Condition for bright ring is, $2t = (2n-1)\frac{\lambda}{2}$

$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by D/2 the diameter of nth bright ring will be

$$\frac{D_n^{\ 2}}{4R} = \frac{(2n-1)\lambda}{2}$$

$$\therefore \quad D \alpha \sqrt{odd\ natural\ number} \quad \text{(for bright ring)}$$

$$D \alpha \sqrt{natural\ number} \quad \text{(for dark ring)}$$

Thus, the diameters of dark rings are proportional to the square root of natural numbers and diameters of bright rings are proportional to odd natural numbers.

Determination of wavelength of a light source:

Let R be the radius of curvature of a Plano convex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of mth and nth dark rings are respectively.

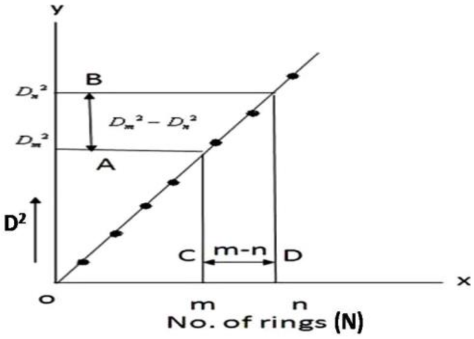


Fig: Plot of D^2 w.r.t. no. of rings

$$D_m^{\ 2} = 4m\lambda R$$

$$D_n^{\ 2} = 4n\lambda R$$

$$D_m^{\ 2} - D_n^{\ 2} = 4(m-n)\lambda R$$

$$\lambda = \frac{D_m^{\ 2} - D_n^{\ 2}}{4(m-n)R}$$

$$R = \frac{D_m^{\ 2} - D_n^{\ 2}}{4(m-n)\lambda}$$

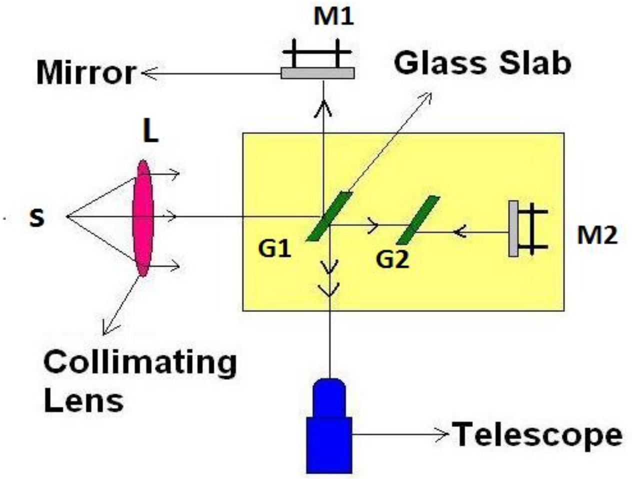
From the graph, $\frac{AB}{CD} = \frac{D_m^2 - D_n^2}{(m-n)}$

The radius R of the Plano convex lens can be obtained with the help of a spherometer. λ can be calculated by substituting all the above values.

Michelson interferometer:

Construction:

- It consists of two excellent optically plane, highly polished plane mirrors M_1 and M_2 which are at right angle to each other. There are two optically flat glass plates G_1 and G_2 of same thickness and of the same material placed parallel to each other. These plates are inclined at an angle 45^0 with the mirrors M1 and M_2. T is a telescope which receives the reflected light from mirrors M_1 and M_2.



Working:

- Monochromatic light from source S after being rendered parallel by a collimating lens L falls on the semi silvered glass plate G_1.
- It is divided into two parts, the reflected ray travels towards mirror M_1 and the transmitted ray travels towards mirror M_2.
- The two rays fall normally on mirrors M_1 and M_2 respectively and are reflected back along their original paths.
- The reflected ray again meets at the semi silvered surface of G_1 and enters a short focus telescope T.
- The two rays entering the telescope are originally derived from the same single beam, hence they are in a position to produce interference fringes in the field of view of the telescope.
- A desired path difference can be introduced between the two reflected rays by moving the mirror M_1.
- It can be noted from figure that the reflected ray passes through G_1 twice where as the transmitted ray does not do so even once.
- Thus in the absence of glass plate G_2 the two paths are not equal.
- To equalise the two paths, glass plate G_2 of same thickness and material as that of G_1 is introduced in the path of transmitted ray.

- Because of this nature, the glass plate G_2 is called compensating plate.
- The interference fringes may be straight, circular, parabolic etc depending upon path difference and angle between mirrors.

Uses of Michelson's interferometer:

Michelson's interferometer has been used for a variety of purposes for example

1. In the determination of wavelength of monochromatic source of light.
2. To determine the difference between the two neighbouring wavelengths or resolution of the spectral lines.
3. In the determination of refractive index and thickness of various thin transparent materials.

Diffraction: The bending of light around the edges of an obstacle is called diffraction. It was first observed by Gremaldy.

When light falls on an obstacle then the corresponding geometrical shadow on the screen should be completely dark. Practically the geometrical shadow consists of bright and dark fringes. These fringes are due to the superposition of bended light waves around the corners of an obstacle. The amount of bending depends upon the size of an obstacle and wavelength of light.

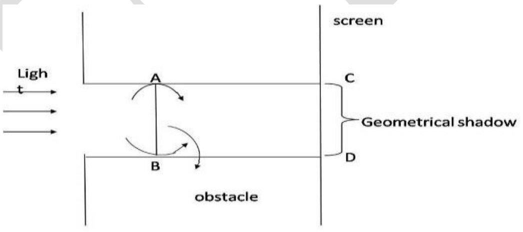


Fig: Diffraction

When light falls on an obstacle whose size is comparable with the wavelength of light then light bends around the edges or corners of an obstacle and enters into the geometrical shadow. This bending of light is known as **diffraction.**

Types of Diffraction: The diffraction phenomena are broadly classified into two types.

1. Fresnel's Diffraction: In this type of diffraction, the source of light and the screen are placed at finite distance. In this, lenses are not necessary to study the diffraction. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

2. **Fraunhofer's Diffraction:** In this type of diffraction, the source and screen are placed at infinite distances. Here we need lenses to study the diffraction. This diffraction can be studied in any direction. In this, the incident wave front is plane.

Difference between Fresnel diffraction and Fraunhofer diffraction:

S. No.	Fresnel diffraction	Fraunhofer Diffraction
1	Source and screen are placed at finite distances.	Source and screen are at placed at infinite distances.
2	No lenses are used.	Lenses are used.
3.	The incident wave front is spherical or cylindrical.	The incident wave front is plane wave front.
4.	The diffraction can be studied in the direction of propagation of light.	The diffraction can be studied in any of propagation of light.

Fraunhofer's diffraction at single slit:

Consider a slit AB of width 'e'. Let a plane wavefront WW' of monochromatic light of wavelength λ propagating normally towards the slit is incident on it. The diffracted light through the slit be focused by means of a convex lens on a screen placed in the focal plane of the lens.

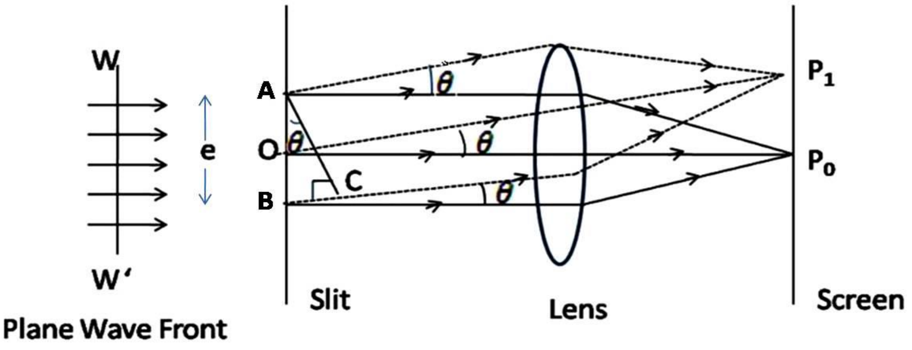


Fig: Fraunhofer Diffraction–Single Slit

According to Huygens-Fresnel, every point on the wavefront in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit i.e., along the direction OP_o are brought to focus at P_o by the lens. Thus, P_o is a bright central image.

The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1 on the screen. Depending on the path difference, the point P_1may have maximum or minimum intensities. In order to find out intensity at P_1, let us draw a parallel AC from A to the light ray at B.

The path difference between secondary wavelets from A and B in direction θ, is given by

$$\Delta = BC = AB\sin\theta = e\sin\theta$$

The relationship between phase difference and path difference is given by

$$Phase\, difference = \frac{2\pi}{\lambda} \times path difference$$

$$= \frac{2\pi}{\lambda} \times e\sin\theta$$

Let the width of the slit is divided into ‘n’ equal parts and the amplitude of the wave from each part is ‘a’. Then the phase difference between any two successive waves is

$$\frac{1}{n}\left(Total\ phase\right) \ = \frac{1}{n}\left(\frac{2\pi}{\lambda}\ esin\theta\right) \ = d \quad \left(say\right)$$

Using the method of vector addition of amplitudes the resultant amplitude R is given by

$$R = \frac{a\, Sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)} = \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

$$= \frac{a\sin\alpha}{\sin\left(\alpha/n\right)} \qquad \text{Where} \quad \alpha = \frac{\pi e\sin\theta}{\lambda}$$

$$= a\frac{\sin\alpha}{\alpha/n} \qquad \left(\because \frac{\alpha}{n}\ is\, very\ small\right)$$

$$= \frac{na\sin\alpha}{\alpha}$$

Let na = A, $\boxed{R = \frac{A\sin\alpha}{\alpha}}$

Resultant intensity (I) is proportional to square of amplitude (R). Therefore, the intensity is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Intensity distribution:

The diffraction pattern consists of central principal maximum for α=0. There are subsidiary or secondary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$. Between secondary maxima, there are minima at positions $\alpha = \pm\pi, \pm2\pi$.

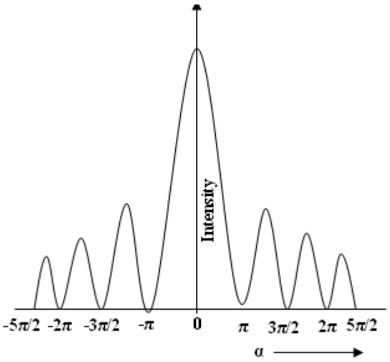


Fig: Intensity Distribution

Diffraction grating:

Diffraction grating is closed placed multiple slits. It consists of very large number of narrow slits side by side separated by opaque spaces. The incident light is transmitted through the slits and blocked by opaque spaces. Such a grating is called transmission grating.

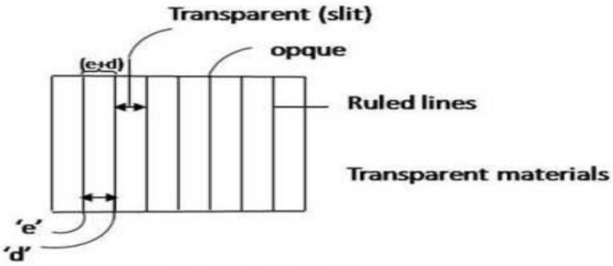


Fig: Diffraction grating

When light passes through the grating, each one of the slit diffracts the waves. All the diffracted waves combine one another producing sharp and intense maxima on the screen.

A plane transmission grating is a plane sheet of transparent material on which opaque lines are made with a diamond point. The space between the rulings is equal and transparent and consists of parallel slits. The combined width of a ruling and a slit is called grating element.

Let 'e' be the width of the line and'd' be the width of the slit. Then (e + d) is known as grating element. If 'N' is the no. of lines per inch on the grating then

$$N(e+d) = 1'' = 2.54cm$$

$$e+d = \frac{2.54}{N}cm$$

When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as diffraction spectrum.

Grating spectrum: The condition to form the principal maxima in a grating is given by $(e+d)\sin\theta = n\lambda$ is called grating equation.

where (e + d) is the grating element and n=1, 2, …...

Instead of monochromatic source of light such as sodium vapour lamp, if white light source such as mercury is used then each diffracted order will have different colours at different angles. For n=1

$$(e+d)\sin\theta_v = \lambda_v \quad (\textit{for violet ray})$$

$$(e+d)\sin\theta_r = \lambda_r \quad (\textit{for red ray})$$

Thus in the spectrum for a grating, there is no overlapping or mixing of colours unlike the spectrum for a prism where different colours overlap. For a grating the angle of diffraction (i.e., angular dispersion) depends on λ and (e + d). Hence if two different gratings of same (e + d) values are chosen, they will produce same dispersion and hence they will be identical.

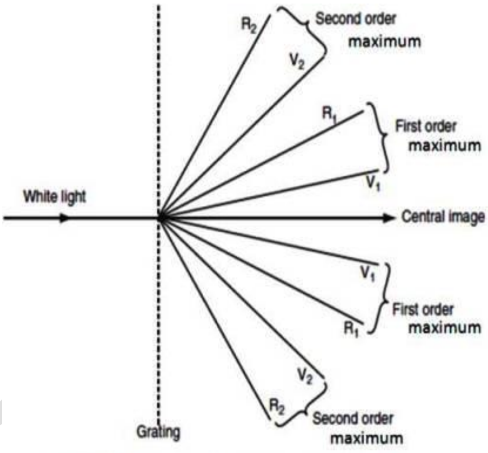


Fig: Grating spectrum for monochromatic source

Maximum no. of orders available with a grating:

The principal maxima in a grating $(e+d)\sin\theta = n\lambda$

$$\frac{1}{e+d} = N$$

Hence $r_1 < r_2, \mu_0 > \mu_e$

For θ=90^0, the maximum possible value of sinθ is 1.

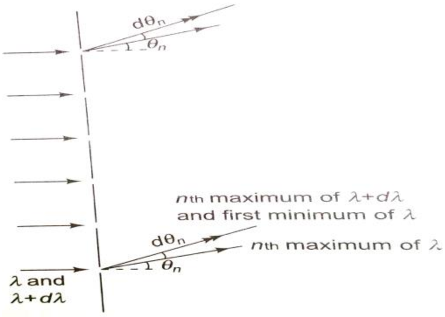
$$nN\lambda \le 1 \quad or \; n \le \frac{1}{N\lambda}$$

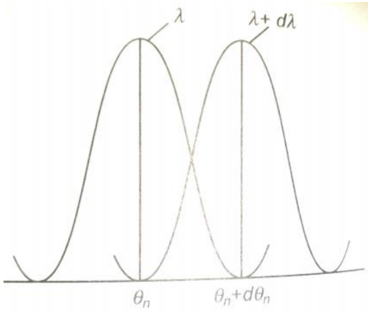
This gives the maximum number of orders possible and n is a integer.

Resolving power of a grating:

The resolving power of a grating represents its ability to form separate spectral lines for wavelengths very close together. It is measured by $\frac{\lambda}{d\lambda}$, where $d\lambda$ is the smallest wavelength difference that can just be resolved at wavelength λ.

Figure:





Analysis:

Let a parallel beam of light of two wavelengths λ *and* $\lambda + d\lambda$ be incident normally on the grating .If the nth principal maximum of λ is formed in the direction θ_n, we will have $(\mathrm{e} + \mathrm{d}) \sin \theta_\mathrm{n} = \mathrm{n}\lambda$ where (e+d) is the grating element.

Now the grating equation for the minima is $N(e+d)sin\,\theta = m\lambda$ where N is the total number of rulings on the grating and m can take all integral values except 0, N,2N,…Nn, because these values of m give respective principal maxima.It is clear from figure that the first minimum of λ adjacent to nth principal maximum of $(\lambda + d\lambda)$ in the direction of increasing θ will be obtained for m= Nn+1.Therefore, for this minimum we have

$$N(e+d)sin\,(\theta_n + d\theta_{n)} = (nN+1)\lambda$$

$$(e+d)sin\,(\theta_n + d\theta_{n)} = (\frac{nN+1}{N})\lambda$$

According to Rayleigh criterion, the wavelengths λ and λ+d λ are just resolved by the grating when the nth maximum of λ+d λ is also obtained in direction θ_n+d θ_n i.e

(e+d)sin(θn + d θn)=n(λ+d λ)………..(4)

Figure shows the overlapping of principal maxima of two patterns

Comparing eqs (3) and(4), we get

$\left(\frac{nN+1}{N}\right)\lambda$ = n(λ+d λ)

nNλ+λ = nNλ+nNdλ

$$\lambda/_{d\lambda} = nN$$

But $(\frac{\lambda}{d\lambda})$ is the resolving power R of the grating. Therefore,

R=nN

$$\mathrm{R} = \frac{N(e+d)\sin\theta n}{\lambda}$$

As expected, the resolving power is zero for the central principal maximum(n=0), all wavelengths being in diffracted in this order.

Unit III INTRODUCTION TO SOLIDS

Free electron theory:

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

It has been developed in three main stages:

1. Classical free electron theory
2. Quantum Free Electron Theory.
3. Zone Theory.

- **Classical free electron theory**: The first theory was developed by Drude & Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.
- **Quantum Free Electron Theory**: In 1928 Sommerfield developed the quantum free electron theory. According to Sommerfield, the free electrons move with a constant potential. This theory obeys quantum laws.
- **Zone Theory**: Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called “Band Theory of Solids”. It gives complete informational study of electrons.

Classical free electron theory:

Even though the classical free electron theory is the first theory developed to explain the electrical conduction of metals, it has many practical applications. The advantages and disadvantages of the classical free electron theory are as follows:

Advantages:

1. It explains the electrical conductivity and thermal conductivity of metals.
2. The valence electrons are freely moving about the whole volume of the metals like the molecules of perfect gas in a container
3. The free electrons moves in random directions and collide with either positive ions or other free electrons. Collision is independent of charges and is elastic in nature
4. The movements of free electrons obey the laws of classical kinetic theory of gases
5. Potential field remains constant throughout the lattice.

Drawbacks:

1. It fails to explain the electric specific heat and the specific heat capacity of metals.
2. It fails to explain Electrical conductivity (perfectly)of semiconductors or insulators.
3. The classical free electron model predicts the incorrect temperature dependence of σ.According to the classical free electron theory ,$\sigma \; \alpha \; T^{-1}$.

Quantum free electron theory of metals:

Advantages:

1. All the electrons are not present in the ground state at 0 K, but the distribution obeys Pauli's exclusion principle. At 0 K, the highest energy level filled is called Fermi-level.
2. The potential remains constant throughout the lattice.
3. Collision of electrons with positive ion cores or other free electrons is independent of charges and is elastic in nature.
4. Energy levels are discrete.
5. It was successful to explain not only conductivity, but also thermionic emission paramagnetism, specific heat.

Drawbacks:

1. It fails to explain classification of solids as conductors, semiconductors and insulators.

Fermi level and Fermi energy:

The distribution of energy states in a semiconductor is explained by Fermi –Dirac statistics since it deals with the particles having half integral spin like electrons. Consider that the assembly of electrons as electron gas which behaves like a system of Fermi particles or fermions. The Fermions obeying Fermi –Dirac statistics i.e., Pouli, s exclusion principle.

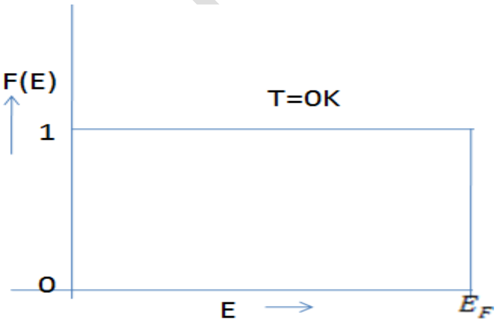
Therefore, the probability function F(E) of an electron occupying an energy level E is given by,

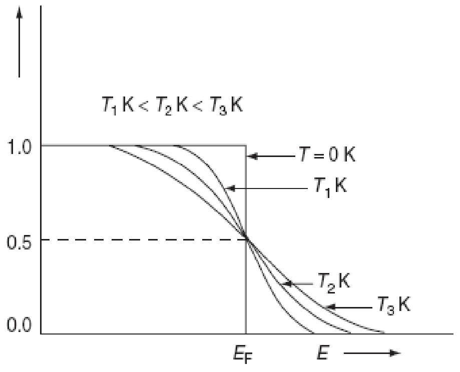
$$F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{KT})} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where E_F known as Fermi energy and it is constant for a system,

K is the Boltzmann constant and T is the absolute temperature.

The probability function F(E) lies between 0 and 1.





Hence there are three possible probabilitities namely

F(E) =1 100% probability to occupy the energy level by electrons.
F(E) =0 No probability to occupy the energy levels by electrons and hence ,it is empty.
F(E) =0.5 50% probability of finding the electron in the energy level.

Case I : Probability of occupation at T= 0K,and $E < E_F$

$$F(E) = \frac{1}{1+\exp\left(\frac{-1}{0}\right)} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e^{\infty}}} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$$

Therefore F(E) =1,as per above ,clearly indicates that at T=0K, the energy level below the Fermi energy level E_F is fully occupied by electrons leaving the upper level vacant. Therefore, there is 100% probability that the electrons to occupy energy level below Fermi level.

Case II: Probability of occupation at T= 0K, and $E > E_F$

Then

$$F(E) = \frac{1}{1+e^{\frac{1}{0}}} = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

i.e., all levels below E_F are completely filled and al levels above E_F are completely empty. As the temperature rises F (E).

Case III: Probability of occupation at T= 0K, and $E = E_F$

$$F(E) = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

The above condition states that, T=0K, there is a 50% probability for the electrons to occupy Fermi energy.

Fermi energy: It is the energy of state at which the probability of electron occupation is ½ at any temperature above 0K. It separates filled energy states and unfilled energy states. The highest energy level that can be occupied by an electron at 0 K is called Fermi energy level

Fermi level: It is a level at which the electron probability is ½ at any temp above 0K (or) always it is 1 or 0 at 0K.

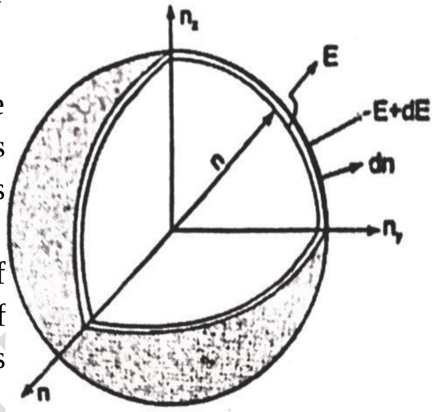
Density of States (DOS):

The number of electrons per unit volume in an energy level at a given temperature is equal to the product of density of states (number of energy levels per unit volume) and Fermi Dirac distribution function (the probability to find an electron).

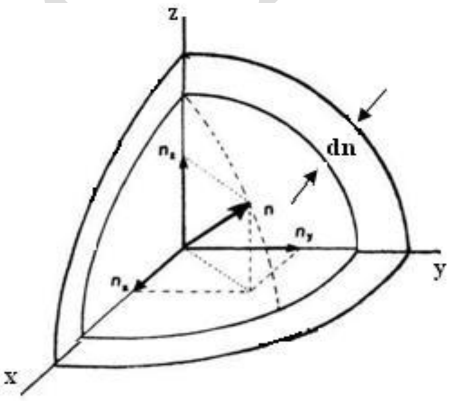
$$n_c = \int g(E) \times f(E)\, dE \ldots\ldots.\quad (1)$$

where n_c is the concentration of electrons, g (E) is the density of states & F(E) is the occupancy probability.

The number of energy states with a particular energy value E is depending on how many combinations of quantum numbers resulting in the same value n.



To calculate the number of energy states with all possible energies, we construct a sphere in 3D- space with 'n' as radius and every point (n_x, n_y and n_z) in the sphere represents an energy state.
As every integer represents one energy state, unit volume of this space contains exactly one state .Hence; the number of states in any volume is equal to the volume expressed in units of cubes of lattice parameters). Also, $n^2 = n_x^2 + n_y^2 + n_z^2$



Consider a sphere of radius n and another sphere of radius n+dn with the energy values are E and (E+dE) respectively.
Therefore, the number of energy states available in the sphere of radius 'n' is
by considering one octant of the sphere

(Here, the number of states in a shell of thickness dn at a distance 'n' in coordinate system formed by n_x, n_y and n_z and will take only positive values ,in that sphere $\frac{1}{8}$ of the volume will satisfy this condition).

The number of energy states within a sphere of radius (n+dn) is

$$\frac{1}{8}\left(\frac{4\pi}{3}\right)(n+dn)^3$$

Thus the number of energy states having energy values between E and E+dE is given by

$$g(E)dE = \frac{1}{8}\left(\frac{4\pi}{3}\right)(n+dn)^3 - \frac{1}{8}\left(\frac{4\pi}{3}\right)n^3$$

$$= \frac{1}{8}\left(\frac{4\pi}{3}\right)\lfloor(n+dn)^3 - n^3\rfloor$$

$$= \frac{\pi}{6}(3n^2dn) = \frac{\pi}{2}\, n^2\, dn$$

compared to 'dn', dn^2 and dn^3 are very small .

Neglecting higher powers of dn

$$g(E)dE = \frac{\pi}{2}\, n^2\, dn \ldots\ldots..(2)$$

The expression for n^{th} energy level can be written as ,

$$E = \frac{n^2h^2}{8mL^2} \qquad or, \qquad n^2 = \frac{8mL^2E}{h^2} \ldots\ldots..(3)$$

$$\Longrightarrow n = \left(\frac{8mL^2E}{h^2}\right)^{\frac{1}{2}} \ldots\ldots\ldots. \quad (4)$$

Differentiating eq. (3):

$$2n\, dn = \frac{8mL^2}{h^2}\, dE \qquad => \; dn = \frac{1}{2n}\left(\frac{8mL^2}{h^2}\right) dE$$

∵ by substituting 1/n value in dn,

$$n = \frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{\frac{1}{2}} \quad \frac{dE}{E^{\frac{1}{2}}} \ldots\ldots\ldots(5)$$

Substitute n^2 and dn from eq. (3) and (5), we get

$$g(E)dE = \frac{\pi}{2}\left(\frac{8mL^2}{h^2}\right) E\;\; dE\; \times \frac{1}{2}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} \frac{dE}{E^{1/2}}$$

$$g(E)dE = \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}\, dE \quad \ldots\ldots\ldots(6)$$

According to Pauli's Exclusion Principle, two electrons of opposite spin can occupy each energy state

Equation (6) should be multiplied by 2

$$g(E)dE = 2 \times \frac{\pi}{4}\left(\frac{8mL^2}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}\, dE$$

After mathematical simplification, we get g

$$(E)dE = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}}\, E^{\frac{1}{2}} L^3\, dE$$

The density of energy states g(E) dE per unit volume is given by, ∵ $L^3 = 1$

$$\boxed{g(E)dE = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}}\, E^{\frac{1}{2}}\, dE}$$

Bloch Theorem:

According to free electron model, a conduction electron in metal experiences constant potential. But in real crystal, there exists a periodic arrangement of positively charged ions through which the electrons move. As a consequence, the potential experienced by electrons is not constant but it varies with the periodicity of the lattice.

In zone theory, as per Bloch, potential energy of electrons considered as varying potential with respect to lattice ‘a’.

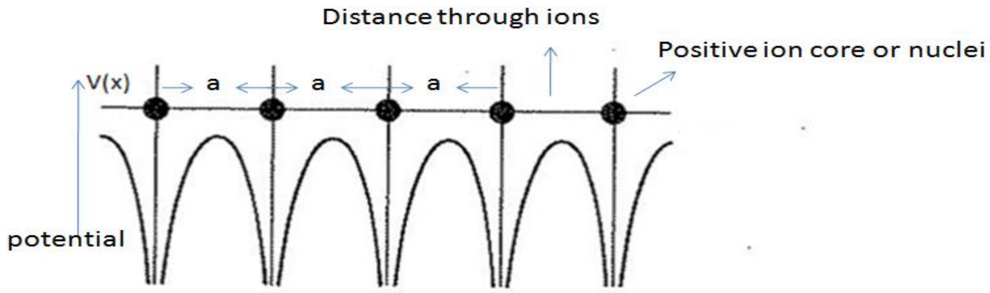


Fig: Variation of potential energy in a periodic lattice.

Let us examine one dimensional lattice as shown in figure. It consists of array of ionic cores along X-axis. A plot of potential V as a function of its position is shown in figure.

From graph:

At nuclei or positive ion cores, the potential energy of electron is minimum and in-between nuclei, the P.E.is considered as maximum w.r.to. Lattice constant ‘a’.

This periodic potential V(x) changes with the help of lattice constant a, $V(x) = V(x + a)$ (‘a’ is the periodicity of the lattice)

To solve, by considering Schrodinger’s time independent wave equation in one dimension,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - V(x)]\psi = 0 \dots\dots \quad (1)$$

Bloch’s 1D solution for Schrodinger wave equation (1) $\psi_k(x) = u_k(x) exp(ikx) \dots\dots\dots (2)$

where $u_k(x) = u_k(x + a)$

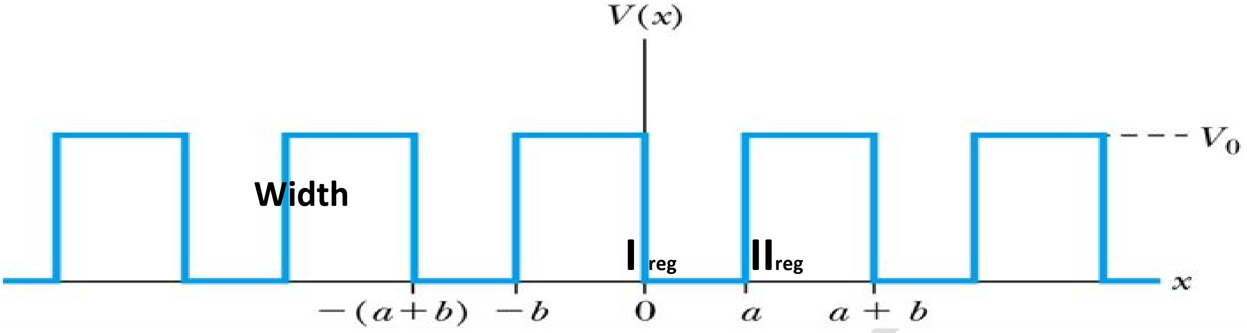
Here $u_k(x)$ -periodicity of crystal lattice, modulating function, k- propagation vector $= \frac{2\pi}{\lambda}$ e^{ikx} is plane wave.

By applying eq.n (2) to eq.n (1),it is not easy to solve Schrodinger wave equation and Bloch cannot explains complete physical information about an e^- in periodic potential field. Then Kronig Penny model was adopted to explains the electrical properties of an e^-.

Kronig-Penney model:

Kronig –penny approximated the potentials of an e^-s inside the crystal in terms of the shapes of rectangular steps as shown, i.e.square wells is known as Kronig Penny model.

i.e. The periodic potential is taken in the form of rectangular one dimensional array of square well potentials and it is the best suited to solve Schrodinger wave equation.



It is assumed that the potential energy is zero when x lies between 0 and a, and is considered as I region. Potential energy is V_0 , when x lies between –b and 0. And considered as II region.

Boundary conditions:

$V(x) = 0$, where x lies between $0 < x < a$ – I region
$V(x) = V_0$, where x lies between $-b < x < 0$ – II region

This model explains many of the characteristic features of the behavior of electrons in a periodic lattice.

The wave function related to this model may be obtained by solving Schrodinger equations for the two regions,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0, \text{for } 0 < x < a \ \ with \ \ V(x) = 0 \dots\dots\dots\dots (1)$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0, \text{for } -b < x < 0 \ \ \text{with} \ \ V(x) = V_0 \dots\dots\dots (2)$$

Again,

$$\frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0 \dots\dots..(3) \qquad \text{where } \alpha^2 = \frac{2mE}{\hbar^2} \text{ and } \alpha = \frac{2\Pi}{h}\sqrt{2mE}$$

$$\frac{d^2\psi}{dx^2} - \beta^2 \psi = 0 \dots\dots.(4) \qquad \text{where } \beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

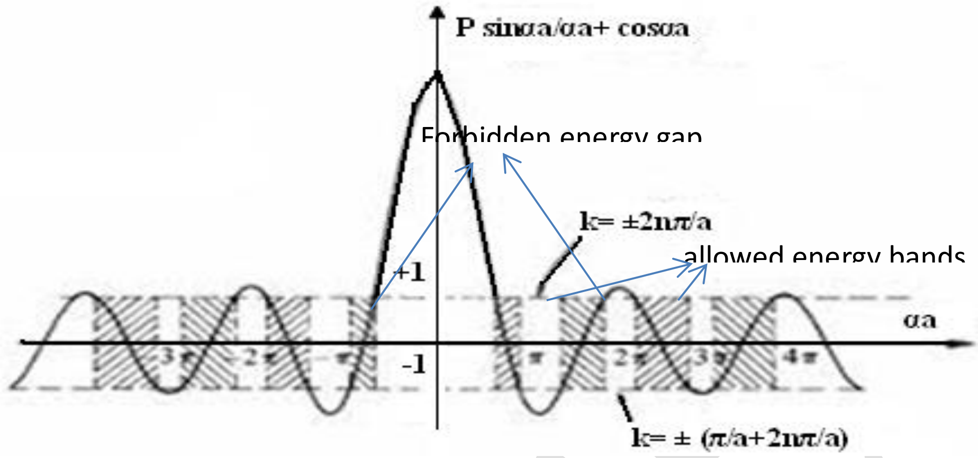
The solution of these equations from Bloch theorem, $\psi_k(x) = u_k(x)exp(ikx)$.

From figure, square well potentials, if V_0 increases, the width of barrier ‘w’ decreases, if V_0 decreases the width of barrier w increases. But the (product) barrier strength $V_0 w$ remains constant.To get this, differentiating above Schrodinger wave equations 3 & 4 w.r.to x, and by applying boundary conditions of x (w.r.to their correspondingΨ), to known the values of constants

A, B of region -I, C,D-for reg-II,we get mathematical expression (by simplification)

$$\boxed{coska = P\frac{sin\alpha a}{\alpha a} + cos\alpha a}$$

where,



$$P = \frac{4\pi^2 ma}{h^2} V_0 w \qquad \text{and} \qquad \alpha = \frac{2\pi}{h}\sqrt{2mE}$$

P-varying term, known as scattering power.

Interpretation of the conditions under which solutions (eq. 2) are possible energy bands: Consider the plot of eq. (2)

1. The L.H.S is a cosine term which varies between the limits -1 and +1, and hence the R.H.S also varies between these limits. It means energy is restricted within -1 to +1 only.

2. If the energy of e^- lies between -1 to +1, are called **allowed energy bands** and it is shown by shaded portion in energy spectrum. This means that 'αa' can take only certain range of values belonging to allowed energy band.
3. As the value of αa increases, the width of the allowed energy bands also increases.
4. If energy of e^-s not lies between -1 to +1 are known as **forbidden energy bands** and it is decreases w.r.to increment of αa.
5. Thus, motion of e^-s. in a periodic lattice is characterized by the bands of allowed & forbidden energy levels.

Case 1:

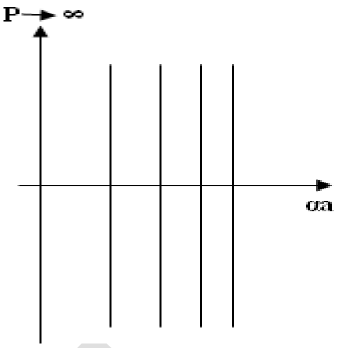
1. P→∞

   If P→∞, the allowed band reduces to a single (line) energy level, gives us steeper lines.

We have

$$coska = P\frac{sin\alpha a}{\alpha a} + cos\alpha a$$

$$coska \ \left(\frac{\alpha a}{p}\right) = \sin \alpha a + cos\alpha a\left(\frac{\alpha a}{p}\right)$$



$$P \rightarrow \infty \ ,\frac{1}{\infty} = 0 \qquad then \quad sin\alpha a = 0$$

$$sin\alpha a = \sin n\pi$$

$$\alpha a = n\pi$$

$$\alpha^2 a^2 = n^2\pi^2$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2}$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2\pi^2 \mathrm{h}^2}{2ma^2\ 4\pi^2}$$

$$E = \frac{n^2\mathrm{h}^2}{8ma^2} \quad , here \quad a \ \ is\ lattice\ constant$$

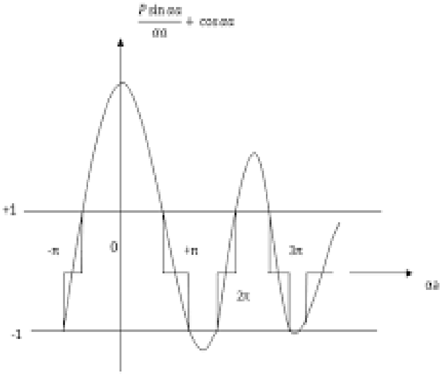
It means, it (zone theory) supports quantum free electron theory.

Case 2:

2. P→0,

We have

$$\alpha a = ka$$



$$\alpha = k$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{\hbar^2} = (\frac{2\pi}{\lambda})^2 = \frac{4\pi^2}{\lambda^2}$$

$$E = \frac{4\pi^2\hbar^2}{2m\lambda^2}$$

$$E = \frac{4\pi^2\mathrm{h}^2}{2m\lambda^2\ 4\pi^2}$$

$$E = \frac{1}{2}\ mv^2$$

It gives us kinetic energy of an electron. It means zone theory supports classical free electron theory at this situation electron completely free electron not bounded with allowed and forbidden gaps(and no energy level exists).

Thus by varying P from 0 to ∞, we find that the completely free electron(s) becomes completely bound to Brillouin Zone**.**

Brillouin Zone (E-K curve):

The total energy of free electron is given by

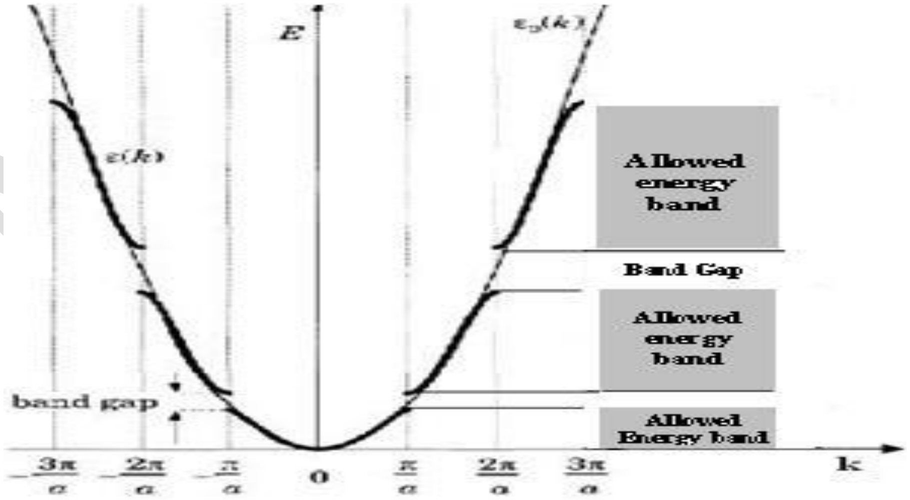
$$E = \frac{\hbar^2 k^2}{2m} = \frac{P^2}{2m}$$

The Brillouin zone is a representation of permissive values of k of electrons in one, two or three dimensions. It is the energy spectrum of an electron moving in presence of a periodic potential field and is divided into allowed energy regions (allowed zones) or forbidden energy gaps (forbidden zones).

The relation between energy and wave number for an electron in one dimensional lattice can be understood by eq. (2). R.H.S of eq. (2) becomes ±1 for values of k=±nπ/a. These values actually define the boundaries of the allowed zone.

Allowed energy values lie in the region k=-π/a to =+π/a. This zone is called the first Brillouin zone.

After a break in the energy values, called forbidden energy band, we have another allowed zone spread from k=-π/a to -2π/a and +π/a to +2π/a. This zone is called the second Brillouin zone. Similarly, higher Brillouin zones are formed.



Concept of effective mass of electron:

When an electron in a periodic potential of lattice is accelerated by an known electric field or magnetic field, then the mass of the electron is called effective mass and is represented by $m*$.

To explain, let us consider an electron of charge ‘e’ and mass ‘m’ moving inside a crystal lattice of electric field E.

Then by taking known expression F=ma, can be considered here as $F = m * a \quad \dots\dots..(1)$

The acceleration$a = \frac{eE}{m}$ is not constant in the periodic lattice but varies due to the change in electronic mass.

If free electron under wave packet, the group velocity V_g corresponding to the particle’s velocity can be written as

$$v_g = \frac{d\omega}{dk} = 2\pi\frac{dv}{dk} = \frac{2\pi}{h}\frac{dE}{dk} = \frac{1}{\hbar}\frac{dE}{dk} \quad \dots .... \;(2) \quad where\ E = hv$$

The rate of change of velocity is known as

$$Acceleration, a = \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk\,dt}$$

$$a = \frac{1}{\hbar}\frac{dE}{dt}\frac{\mathrm{dk}}{\mathrm{dk}} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} \dots ..(3)$$

$$From\ quantum\ mechanics\ relation\ , p = \hbar k\ and\ F = \frac{dp}{dt} => \frac{dk}{dt} = \frac{F}{\hbar} \quad \dots ....(5)$$

$$\therefore a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{F}{\hbar} => F = \frac{\hbar^2 a}{\left(d^2E/_{dk^2}\right)} \quad \dots \dots (6)$$

$$Comparing\ eqs.(2)\ \&\ (6), we\ get\ m^* = \frac{\hbar^2}{\left(d^2E/_{dk^2}\right)} \dots \dots (7)$$

Hence effective mass is determined by $\left(d^2E/_{dk^2}\right)$.

Significance of Effective Mass of electron:

a) **Infinite Effective Mass**: Infinite or very large effective mass means that it is very difficult for an external field to move the electron due to strong binding force between the electron and the lattice.

b) **Negative Effective Mass**: When electron approaches the positive ion core it experiences large retarding potential i.e., negative acceleration. This negative mass concept is referred to as hole.

Origin of energy band formation in Solids:

The existence of continuous bands of allowed energies can be understood starting with the atomic scale. The electrons of a single

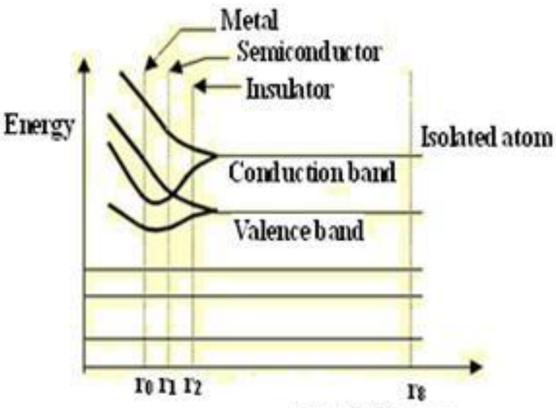


Fig: Formation of energy bands when atoms are closer isolated atom occupy atomic orbitals, which form a discrete set of energy levels. When two identical atoms are brought closer, the outermost orbits of these atoms overlap and interact. When the wave functions of the electrons of different atoms begin to overlap considerably, the energy levels corresponding to those wave functions split.

If more atoms are brought together more levels are formed and for a solid of N atoms, each of the energy levels of an atom splits into N energy levels. These energy levels are so close that they form an almost continuous band. The width of the band depends upon the degree of overlap of electrons of adjacent atoms.

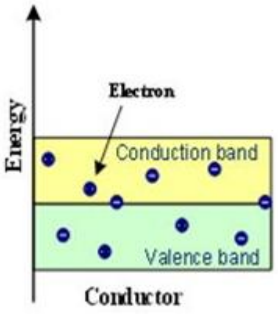
As a result of the finite width of the energy bands, gaps are essentially leftover between the bands called forbidden energy gap. The bands have different widths and therefore two adjacent bands may simply not be wide enough to fully cover the range of energy. For example, the bands associated with core orbital (such as 1s electrons) are extremely narrow due to the small overlap between adjacent atoms. As a result, there tend to be large band gaps between the core bands. Higher bands involve larger and larger orbital with more overlap, becoming progressively wider and wider at high energy so that there are no band gaps at high energy.

Classification of Materials- Metals, Semiconductors & Insulators

Based on the energy band diagram materials or solids are classified as follows:

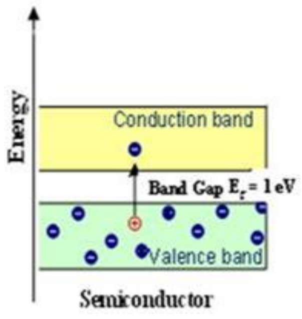
Metals:

In this kind of materials, there is a partially filled band, called the conduction band, just above another uppermost filled band, called valence band. There is no forbidden gap between the valence band and conduction band as they overlap each other. Due to the overlapping of the two bands there is an easy transition of electrons from one band to another band takes place. The electron present in the conduction band is called conduction electrons whereas the electron in the valence band is called valence electrons. Examples are: All metals (Na, Mg, Al, Cu, Ni etc)

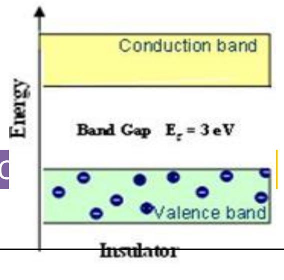


Semiconductors:

In semiconductors, there is a band gap exists between the completely filled valence band and completely empty conduction band. The forbidden energy gap is around $E_g \leq 1$ eV. Due to thermal vibrations within the solid, some electrons gain enough energy to overcome the band gap (or barrier) and behave as conduction electrons. Examples are: Silicon and Germanium.



Insulators:



In insulators, the forbidden energy gap between the completely filled valence band and completely empty conduction band is comparatively large i.e., $E_g \leq 3$ eV. Due to large energy gap, the conduction is not possible. Depending upon the magnitude of band gap insulators are classified into dielectrics.

UNIT IV DIELECTRICS & MAGNETIC PROPERTIES

Introduction:

Dielectrics are non-metallic materials of high specific resistance and have negative temperature coefficient of resistance. Glass, Mica, water, Bakelite are examples of some dielectrics.

The behavior of dielectric can be explained on the basis of atomic view. In the case of dielectric materials, the charged particles are bound strongly to the constituent atoms. When the external electric field is applied to the dielectric medium the charged particles inside it may be able to move to & fro from their equilibrium position, but they do not leave the neighbouring atoms. This is the case of an ideal dielectric medium. But, real dielectric does posses a feeble conductivity which is 10^{20} times smaller than the conductivity of a conductor. For all practical purposes, we can assume dielectrics to be a bad conductor.

These are generally insulating materials. These are of two types.

a) Non-polar Di-electric material b) Polar Di-electric material

Non –polar di-electrics:

These materials generally consists of diatomic molecules composed of atoms, each atom consists of positive nucleus and cloud of negative charge of charge of– q. If E is zero, all centers of positive charges and all centers of negative charges coincide with each other. If E is present, then centers of positive and negative charges move apart by a small distance of ($\mathbf{10^{10}}$ m), then the molecules and atoms said to be polarized.

Polar –dielectrics:

These generally consist of more than two atoms composed of different atoms. Even E is zero, also they have dipole moments in random directions. If E is present, there dipole moments tend to align in the field direction. So that material has become resultant dipole moment. Due to this centers of positive and negative are separated with a very small distance which produces dipole moment (**P**).

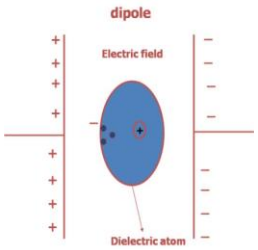
Fundamental Definitions:

Permitivity:

Permittivity, also called electric permittivity, is a constant of proportionality that exists between electric displacement and electric field intensity. This constant is equal to approximately 8.85 x 10^{-12} farad per meter (F/m) in free space (a vacuum). In other materials it can be much different, often substantially greater than the free-space value, which is symbolized $_o$.

Electric dipole:

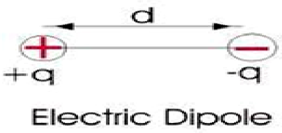
Every atom can be considered as a dipole since there is a positively charged central nucleus around which, there is equal number of negatively charged electrons revolving around the nucleus.



An electric dipole is a separation of positive and negative charges. The simplest example of this is a pair of electric charges of equal magnitude but opposite sign, separated by some (usually small) distance.

Electric dipole moment: It is defined as the product of the charge and the distance between the charges. Mathematically it is given by

$$\mu = q \times d \quad \text{Coulomb-meter}$$



Dielectric constant (Єr): -

The dielectric characteristics of a material are determined by the dielectric constant or relative permittivity ($Є_r$). Dielectric constant is defined as the ratio of the permittivity of the medium(ϵ) to the permittivity of free space (ϵ_0) i.e.,

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Since it is a ratio of same quantity, $Є_r$ has no unit. It is a measure of polarization in the dielectric material.

Electric polarization: -

The process of producing electric dipoles which are oriented along the field direction is called polarization in dielectrics. If we place an atom inside an electric field (E), the centre of positive charge will be displaced in the direction of the electric field E and the centre of negative charge will be displaced in a direction opposite to the field E. Thus dipole will form.

Polarizability (α): -

When the strength of the electric field E is increased the strength of the induced dipole μ also increases. Thus the induced dipole moment is proportional to the intensity of the electric field i.e.,

$$\mu = \alpha E$$

where α is constant of proportionality is called Polarizability.

It can be defined as induced dipole moment per unit electric field

$$\alpha = \frac{\mu}{E}$$

$$units: farad - m^2$$

Polarization vector ($\vec{P}$):

The dipole moment per unit volume of the dielectric material is called polarization vector ($\vec{P}$). If $\bar{\mu}$ is the average dipole moment per molecule and N is the no. of molecules per unit volume then the polarization vector.

$$\vec{P} = N\bar{\mu}$$

Note: The dipole moment per unit volume of the solid is the sum of all the individual dipole moments within that volume.

Electric flux density or Electric displacement (D): -

The electric flux density or electric displacement 'D' at a point in a material is given by

$$D = \epsilon_r \epsilon_0 E$$

Where E is Electric field strength;
ϵ_0 is dielectric constant or permittivity of free space (vacuum) &
ϵ_r is the dielectric constant or relative permittivity of the material

As the polarization measures the additional flux density arising from the presence of the material as compared to free space, it has the same units as D and is related to it as

$$D = \epsilon_0 E + P$$

$$\text{But } D = \epsilon_r \epsilon_0 E$$

$$\therefore \quad \epsilon_r \epsilon_0 E = \epsilon_0 E + P$$

$$\boxed{P = \epsilon_0 (\epsilon_r - 1) E}$$

Electric flux density 'D' is similar to magnetic induction B in magnetism.

Electric susceptibility(χ_e): -

The polarization vector $\overline{P}$ is proportional to the total electric flux density E and is in the same direction of E , then the polarization vector can be written as

$$P = \epsilon_0 \chi_e E \quad \text{where } \chi_e \text{ is electric susceptibility}$$

$$(or) \quad \chi_e = \frac{P}{\epsilon_0 E} = \frac{\epsilon_0 (\epsilon_r - 1) E}{\epsilon_0 E} = (\epsilon_r - 1)$$

It has no unit.

Relative dielectric constant: -

The polarization (or) the relative dielectric constant can be measured using parallel plate capacitor. When a voltage V is applied to a capacitor with the plates separated by vacuum charge develops on the plates.

Therefore, the capacitance C of the capacitor is a measure of this charge and is given by

$$C = \frac{\epsilon_0 A}{d}$$

where, ϵ_0 is permittivity of free space
A is area of plate
D is separation between the plates

If dielectric material is inserted between the plates, charge on the plates increase due to polarization in the material.

$$\therefore C^1 = \frac{\epsilon_0 \epsilon_r A}{d}$$

$\therefore$ Dielectric constant can be expressed as

$$\boxed{\epsilon_r = \frac{C^1}{C}}$$

Polarization processes: -

Polarization occurs due to the atomic mechanisms in a solid material. When a dielectric specimen is placed inside a d.c. electric field, the following four types of polarization are possible.

1. Electronic polarization
2. Ionic polarization
3. Orientational /Dipolar polarization and
4. Space charge polarization

Electronic polarization:

The displacement of the positively charged nucleus and the (negative) electron of an atom in opposite directions due to the applied electric field results in electronic polarization.

On application of external electric field E, the electron cloud around the nucleus readily shifts towards the positive end of the field. Because of this dipole moment is created within each atom due to the separation of positive and negative charges. This separation or shift is proportional to field strength (E).

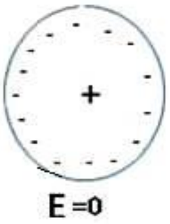
As the dipole moment is the product of charge and the separation between the charges,

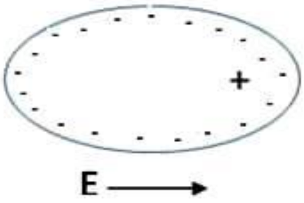
Induced dipole moment, $\mu \propto E$

$$\mu = \alpha_e E$$

α_e is constant of proportionality called electronic polarizability and it is independent of temperature. It occurs only at optical frequencies (10^{15}Hz)

<u>Calculation of Electronic polarizability</u>:

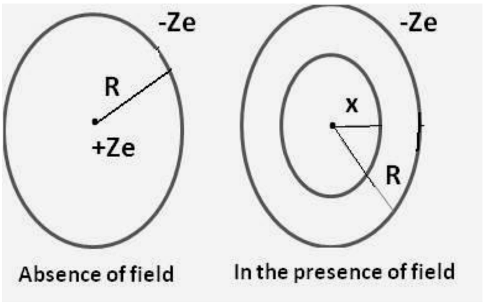




Electronic polarization takes place in all dielectrics due to displacement of charge centers of electron cloud and nucleus of atoms in the presence of an applied electric field as shown in above figure.

By taking classical model of an atom, the nucleus of charge Ze is surrounded by an electron cloud of charge –Ze distributed in a sphere of radius R.

Then the charge density (Z – atomic number) is given by



When an external electric field E is applied, the nucleus and electrons experience Lorentz forces of magnitude ZeE in opposite direction. Because of this, the nucleus and electron cloud are pulled apart.

$$\text{Lorentz force} = -ZeE \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Let 'x' be the displacement under this condition. Here, nucleus is much heavier than the electron cloud; it is assumed that only the electron cloud is displaced when the external field is applied.

Due to the separation between the nucleus and electron cloud, a coulomb force of attraction is developed and is given by

$$Coulomb\ force = Ze \times \frac{chrge\ enclosed\ in\ the\ sphere\ of\ radius\ x}{4\pi\epsilon_0 x^2} \ldots..(3)$$

$$The\ charge\ enclosed\ = \frac{4}{3}\pi x^3 \rho$$

$$= \frac{4}{3}\pi x^3 \left(\frac{-Ze}{\frac{4}{3}\pi R^3}\right) = \frac{4}{3}\pi x^3 \left(\frac{-3}{4}\right)\frac{Ze}{\pi R^3} = \frac{-Zex^3}{R^3} (\because eqn\ 1) \ldots\ldots\ldots(4)$$

By substituting eq^n (4) in eq^n (3)

$$Coulomb\ force\ = Ze \times \left(\frac{-Zex^3}{R^3}\right) \times \frac{1}{4\pi\epsilon_0 x^2} = \frac{-Z^2e^2x}{4\pi\epsilon_0 R^3} \ldots\ldots..(5)$$

At thermal equilibrium the Lorentz force is equal to the Coulomb force of attraction i.e.,

$$-ZeE = \frac{-Z^2e^2x}{4\pi\epsilon_0 R^3} \qquad [\because eqn\ (2) \& eqn\ (5)]$$

$$or\ \boxed{x = \frac{4\pi\epsilon_0 R^3 E}{Ze}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

Thus, displacement of electron cloud is proportional to applied field

$$x \propto E$$

∴ The electric charges +Ze and –Ze are separated by a distance x under the influence of E constituting induced dipoles (moment).

Induced electric dipole moment,

$$\mu_e = Zex [\because eqn\ (6)]$$

$$= Ze.\frac{4\pi\epsilon_0 R^3 E}{Ze}$$

$$\mu_e = 4\pi\epsilon_0 R^3 E$$

i.e. $\mu_e = \alpha_e E$

Where $\alpha_e = 4\pi\epsilon_0 R^3$ is called electronic polarizability

It is the dipole moment per unit volume & is independent of temperature.

$$P_e = N\mu_e$$

$$= N\alpha_e E$$ (Where, N is the no. of atom/m^3)

`

$$P_e = \epsilon_0 E(\epsilon_r - 1) = N\alpha_e$$

$$\boxed{\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0}}$$

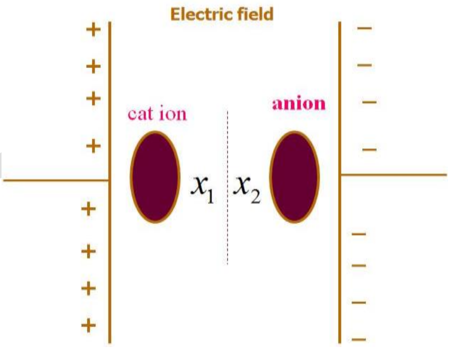
$$Hence, \boxed{\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}}$$

Ionic polarization:

The ionic polarization is due to the displacement of cation and anions in opposite direction and occurs in an ionic solid under the application of an external electric field.

To explain, consider an electric field applied in the positive x direction then the positive ions move to the right by x$_1$ and the negative ions move to left by x$_2$. If we assume each unit cell has one cation and one anion, the resultant dipole moment per unit cell due to ionic displacement is given by

If β$_1$ and β$_2$ are restoring force constants of cation and anion, the Newtonian force is given by



Restoring force constants depends upon the mass of the ion and angular frequency of the molecule in which the ions are present

$$\therefore x_1 = \frac{eE}{m\omega_0^2} \;\&\; x_2 = \frac{eE}{M\omega_0^2}$$

where m- mass of the positive ion & M-mass of the negative ion

The distance between the two ions is given by

$$x_1 + x_2 = \frac{eE}{\omega_0^2}(\frac{1}{M} + \frac{1}{m})$$

Now the dipole moment can be written as

$$\mu = e(x_1 + x_2) = \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right)[\because eqn(1)]$$

Finally ionic polarizability is

$$\boxed{\alpha_i = \frac{\mu}{E} = \frac{e^2}{\omega_0^2}(\frac{1}{M} + \frac{1}{m})}$$

and it is independent of temperature. This polarization occurs at frequency 10^{13} Hz (IR).It is a slower process compared to electronic polarization.

Internal field/local field in solids [Lorentz Method]:

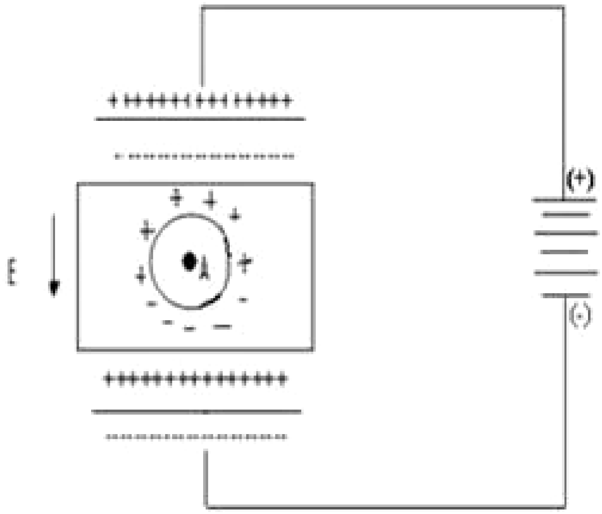
In dielectric solids, the atoms or molecules experience not only the applied external electric field but also the electric field produced by the dipoles in it.

Now a dielectric material is placed between the two parallel plate capacitor and there be an imaginary spherical cavity around the atom 'A' inside the dielectric. By taking that the radius of the cavity is large compared to the radius of the atom. Now, the internal field at the atom site 'A' can be considered to be made up of four components namely E_1,E_2,E_3 and E_4

Field E_1: -

E_1 is the field intensity at A due to the charge density on the plates of the capacitor

From the field theory,



FieldE_2: -

E_2 is the field intensity at A due to charge density induced on the two sides of the dielectric

Therefore

$$E_2 = \frac{-P}{\epsilon_0} \ldots \ldots \ldots \ldots \ldots . . (2)$$

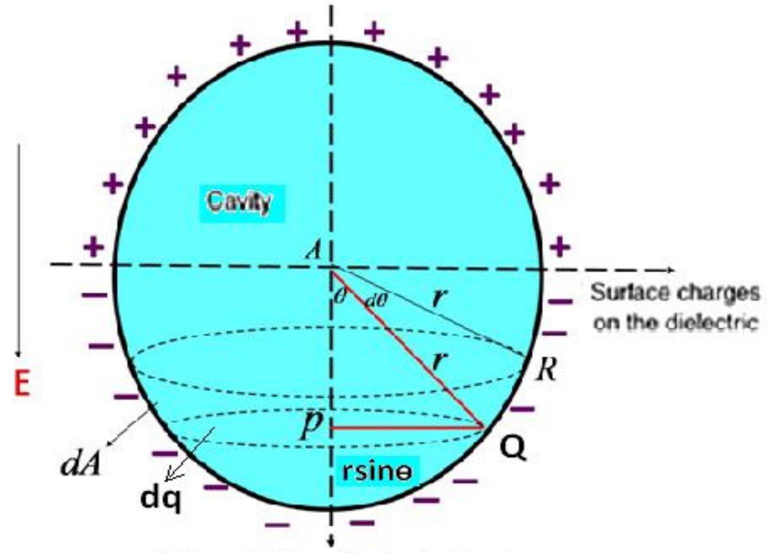
FieldE_3:-

E_3 is the field intensity at A due to other atoms contained in the cavity. If we assume a cubic structure, then

$$E_3 = 0 \dots\dots\dots\dots\dots\dots (3) \quad \text{because of symmetry.}$$

<u>FieldE_4</u>:

E_4 is the field density due to polarization charges on the surface of the cavity and it is calculated by Lorentz. The enlarged view of the cavity is shown.



Enlarged view of spherical region

If dA is the surface area of the sphere of radius r lying between ө and ө+dө where ө is the direction with reference to the direction of the applied force and dq is the charge enclosed in this surface area

Then (from fig) $dA = 2\pi(PQ)(QR) \dots\dots\dots\dots\dots\dots. (4)$

Here from Δ APQ,

$$sin\theta = \frac{PQ}{r}$$

$$PQ = rsin\theta \dots\dots\dots\dots\dots (5)$$

And from Δ AQR,

$$d\theta = \frac{QR}{r} \Longrightarrow QR = rd\theta \dots\dots\dots\dots\dots.. (6)$$

Hence, by substituting eqn(5) & eqn (6) in eqn (4)

$$dA = 2\pi(r sin\theta)(r d\theta)$$

$$dA = 2\pi r^2 sin\theta d\theta \ldots\ldots\ldots\ldots\ldots(7)$$

The charge dq on the surface dA is equal to the normal component of the polarization multiplied by the surface area.

$$\therefore dq = p cos\theta \times dA = P(2\pi r^2 sin\theta cos\theta d\theta) \ldots\ldots\ldots(8)$$

Field due to this charge at the centre 'A' is denoted by dE₄ and is obtained by imagining a unit charge at point 'A'

$$dE_4 = \frac{dq \times 1 \times cos\theta}{4\pi\epsilon_0 r^2}$$

$$= \frac{P(2\pi r^2 sin\theta cos\theta d\theta)cos\theta}{4\pi\epsilon_0 r^2} [\because eqn(8)]$$

[cosθ= component along x-direction]

$$dE_4 = \frac{P}{2\epsilon_0} cos^2\theta sin\theta d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

Now the total field E₄ due to the charges on the entire surface of the cavity is given by integrating above equation

$$\int dE_4 = \frac{P}{2\epsilon_0}\int_0^\pi cos^2\theta\, sin\theta d\theta$$

$$E_4 = \frac{P}{2\epsilon_0}\int_0^\pi cos^2\theta\, d(-cos\theta)$$

$$E_4 = -\frac{P}{2\epsilon_0}\left(\frac{cos^3\theta}{3}\right)_0^\pi$$

$$E_4 = -\frac{P}{6\epsilon_0}(-1-1)$$

$$E_4 = \frac{P}{3\epsilon_0} \ldots\ldots\ldots\ldots\ldots.(10)$$

∴The internal field or Lorentz field can be written as

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = \left(E + \frac{P}{\epsilon_0}\right) + \left(\frac{-P}{\epsilon_0}\right) + 0 + \frac{P}{3\epsilon_0} \qquad [\because from\ eqn(1),(2),(3)and(10)]$$

$$E_i = E + \frac{P}{3\epsilon_0} \ldots\ldots\ldots\ldots..(11)$$

eqn (11) is known as internal field (or) Lorentz field.

Clausius – Mosotti relation: -

The relation between the dielectric constant and the polarizability of a polarized elemental dielectric medium is known as Clausius – Mosotti equation. Consider the elemental dielectric medium having cubic structure such that the ionic polarizability α_i and orientational polarizability α_0 are zero i.e,

$$\alpha_i = \alpha_0 = 0$$

But polarization is given by $P = N\alpha_e E_i \ldots\ldots(1)$

$$P = N\alpha_e\left(E + \frac{P}{3\epsilon_0}\right) \ldots\ldots..(2) \qquad (\because by\ substituting\ E_i value\ )$$

$$P\left(1 - \frac{N\alpha_0}{3\epsilon_0}\right) = N\alpha_0 E \quad (or) \quad P = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_0}{3\epsilon_0}\right)} \ldots\ldots...(3)$$

We have,

$$P = E\epsilon_0(\epsilon_r - 1) \ldots\ldots\ldots\ldots.. \qquad (4)$$

Using (3) and (4):

$$P = E\epsilon_0(\epsilon_r - 1) = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\epsilon_0}\right)}$$

By rearranging the above equation

$$1 - \frac{N\alpha_e}{3\epsilon_0} = \frac{N\alpha_e}{\epsilon_0(\epsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\epsilon_0}\left(1 + \frac{3}{\epsilon_r - 1}\right)$$

$$\Rightarrow \frac{N\alpha_e}{3\epsilon_0} = \frac{1}{\left(1 + \frac{3}{\epsilon_r - 1}\right)}$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Eqn (5) is called Clausius – Mosotti equation where N is the no. of atoms by knowing ϵ_r, we can determine α_e

MAGNETIC PROPERTIES

Introduction:

The study of magnetism was started from few centuries ago, with the identification of magnetite **($Fe_3 O_4$**) exhibiting attraction towards iron filings (small pieces of iron).

The materials which strongly attract a piece of iron are known as magnetic materials, or magnet. The magnetic property of materials arises due to the magnetic moment or magnetic dipole of materials. The magnetism of materials is responsible for magnetic moment of materials. Now a day's magnetism is not treated as a separate subject, but as an effect due to electric currents.

When a current flows through a conductor, it produces magnetic moment along the axis of the coil, according to modern theory; circulating charges (current loops) are always associated with magnetic properties.

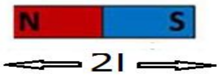
Similarly the electrons revolving around the nucleus leads to an orbital magnetic moment. Due to spin of electrons $\pm 1/2$ spin magnetic moment arises.

These magnetic materials generally bar-magnet, horse- shoe magnet, and needle-type magnet to study the properties and for various applications.

Any material that can be magnetized by application of external magnetic field is called magnetic materials. Electrons revolving around the nuclei in the atom behave like tiny magnetic dipoles giving rise to magnetism in matter.

Basic definitions:

Magnetic dipole: A magnet consists of north and south poles. Any two opposite poles separated by a finite distance constitute a magnetic dipole.

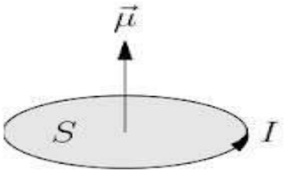


Magnetic moment of a bar magnet: The magnetic moment of magnet is defined as 'moment of couple required to rotate the magnet such that it is perpendicular to the magnetizing field of intensity one or unity.

Mathematically, if 'm' is magnetic pole strength, 2l is length of the magnet then magnetic dipole moment is given as the product of pole strength & distance between the poles.

$$\mu_m = m \times 2l$$

Magnetic moment of a current loop: When an electron current of ' I ' ampere flows through a circular wire of one turn having an area of cross section 'A' , it is said to have a magnetic moment of



Magnetic field: The space surrounding the magnet up to which its attracting influence is felt is known as magnetic field.

Magnetic field strength (H): It is the force experienced by a unit North Pole placed at a given point in a magnetic field.

Units: Gauss (CGS), $N / A - m$ (or) $N / Weber$ $(S.I)$

Magnetization (M): The magnetic moment per unit volume is known as magnetization of the material.

$$M = \mu_{m\,/}\, m3$$

$= induced\ dipole\ moment/volume$; units: A / m

Magnetic susceptibility: It is defined as the ratio of the magnetization to the magnetic field strength.

$$\chi = M / H$$

$(\chi: has\ no\ units\ )$

Magnetic Flux Density: It is defined as the number of magnetic lines of force passing through per area of cross section of magnetic material.

$$\phi = B/A \quad (Units: Weber / m^2 \text{ or } Tesla) \quad (or) \quad B = F/m \; (units: N/A - m)$$

Relative permeability (μ_r):
It is defined as the ratio of permeability of a material to the permeability of free space.

$$\mu_r = {}^{\mu}/_{\mu_0}$$

(or)

$$\mu = \mu_r . \mu_0$$

Ratio between B, H & M:

$$B = \mu H = \mu_0 \mu_r H = \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$
$$= \mu_0 H + \mu_0 H (\mu_r - 1)$$

$$B = \mu_0 H + \mu_0 M \quad [\because M = H(\mu_r - 1)]$$

$$\therefore B = \mu_0 (H + M) \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Relation between χ & μ_r:

$$B = \mu_0 (H + M) \quad (\because eq(1))$$

$$\mu_0 = \frac{B}{H + M} \quad \rightarrow (2)$$

$$But\ H\mu = B \ldots\ldots\ldots\ldots.(3)$$

From (2) & (3) we can write

$$\frac{\mu}{\mu_0} = \frac{(\frac{B}{H})}{\frac{B}{(H + M)}}$$

$$= (H + M) / H = 1 + (M / H)$$

$$\mu_r = 1 + \chi \quad (\because \chi = M / H)$$

Origin of magnetic moment:

Materials are made up of atoms. These atoms consist of a nucleus surrounding which electrons are revolving in their orbits. These orbital motions of e^- establish a magnetic field and hence we get orbital magnetic moment of the electrons. While revolving around the nucleus, the e^-(s) spin about their own axis which results in a magnetic field and we get spin magnetic moment of the electrons. The nucleons in the nucleus also spin, contributing to magnetic field of the nucleus and we get spin magnetic moment of nucleus .i.e., in atoms, the permanent magnetic moments can arise due to the following

(i) The orbital magnetic moment of the electrons.

(ii) The spin magnetic moment of the electrons &
(iii) The spin magnetic moment of the nucleus.

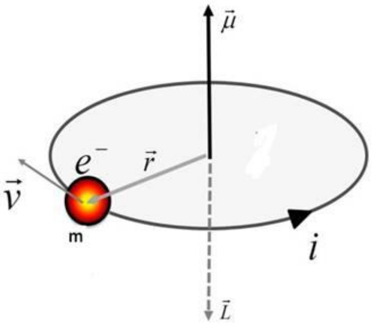
(i)The orbital magnetic moment of the electrons- Bohr Magneton:

Consider an electron of mass ‘m’, charge ‘e’ revolving around the nucleus in a circular orbit of radius ‘r’ in anti-clock wise direction .Let ‘v’ be the linear velocity of e^- and ‘ω’ be the angular velocity of revolving e^-.

The linear frequency of the revolving electron= $\omega/2\pi$ ($\because \omega = 2\pi\nu \Rightarrow \nu = \omega/2\pi$)

$$Time\ period\ of\ revolutions, T = \frac{2\pi}{\omega} \qquad \left(\because\ \nu = \frac{1}{T}\right)$$

The revolving e^-in circular orbit establishes a current given by



The current I establishes a magnetic field around the circular orbit, so that the upper surface acts as south pole &the lower surface acts as north pole .

The area of the orbit is A=πr^2, then the corresponding magnetic moment is given by

$$\mu_{orbital} = I \times A$$

$$= \frac{e\omega}{2\pi} \times \pi r^2$$

$$\frac{e\omega r^2}{2} \times \frac{m}{m} = \frac{e}{2m}(m\omega r^2)$$

$$\therefore \mu_{orbital} = \frac{-e}{2m}\ is\ orbital\ angular\ momentum$$

(Here, -ve sign indicates that the orbital angular momentum and magnetic moment are in opposite direction.

Angular moment of e^-.s associated with orbital quantum number ‘l’ is given by $lh/2\pi$

$$\mu_{orbital} = (-\frac{e}{2m})(\frac{lh}{2\Pi})$$

$$\mu_{orbital} = \gamma\left(\frac{lh}{2\pi}\right)\ \ where\ \gamma = \frac{e}{2m}\ called\ orbital\ gyromagnetic\ ratio$$

$$\therefore \mu_{orbital} = -\left(\frac{eh}{4\pi m}\right) or\ \mu_{orbital} = -\mu_B \times l where\ \ \mu_B = \frac{eh}{4\pi m}$$
$$= 9.27 \times 10^{-24} A\, m^2 called\ Bohr\ magneton\ \&\ l = 1, 2, 3 \ldots\ldots\ldots.$$

Bohr Magneton:

It is the magnitude of the magnetic dipole moment of an orbiting electron which is represented by $'\mu_B'$ (i.e., magnetic dipole moment associated with spin of e^- orbital angular moment)

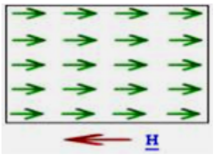
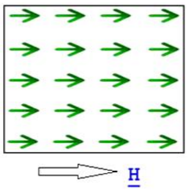
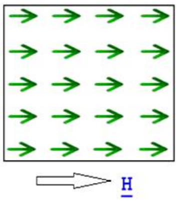
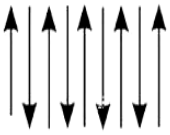
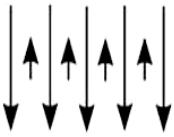
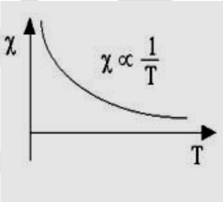
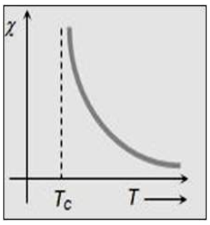
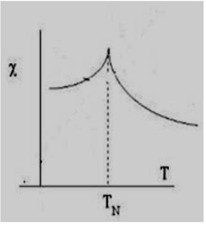
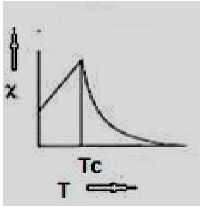
In atomic physics, Bohr magneton is a physical constant

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m}$$

e – Elementary charge; ħ – reduced Planck's constant (Hermitian operator)

Types of Magnetic materials:

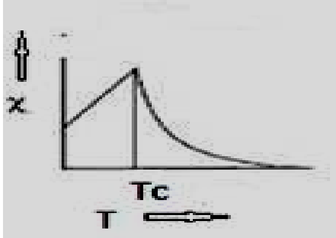
Dia magnetic	**Para magnetic**	**Ferro magnetic**	**Anti ferro magnetic**	**Ferri magnetic**
1. The material which do not have permanent magnetic dipoles is known as Dia magnetic material	The material which have permanent magnetic dipoles is known as Para magnetic material	The material having permanent magnetic dipoles & exhibit spontaneous magnetization is known as Ferro magnetic material.	The materials with anti parallel magnetic moments are known as anti Ferro magnetic material.	The substances consist of anti parallel magnetic moments of different magnitudes are known as Ferri magnetic material.

 2.				
3. Net magnetic moment exists in the presence of magnetic field in opposite direction	Net magnetic moment exists in the direction of applied magnetic field	Large magnetic moment exists in the direction of applied magnetic field	Net magnetic moment exists in the direction of applied magnetic field	Net large magnetic moment exists in the direction of applied magnetic field
4. For this $\mu_r < 1$	For this $\mu_r > 1$	For this $\mu_r >> 1$	For this $\mu_r > 1$	For this $\mu_r >> 1$
5. χ is negative	χ is low & positive	χ is high & positive	χ is low & positive	χ is high & positive.
6. It repels the magnetic lines of forces due to the applied field. $(B_{in} < B_{out})$	It allows the magnetic lines of forces to pass through it. $(B_{in} < B_{out})$	It allows the magnetic lines of forces to pass through it. $(B_{in} >> B_{out})$	It allows the magnetic lines of forces $(B_{in} > B_{out})$	It allows a large number of magnetic lines of forces $(B_{in} >> B_{out})$
7. Intensity of magnetization 'M' is negative	Intensity of magnetization 'M' is positive moderate	Intensity of magnetization 'M' is positive large	Intensity of magnetization 'M' is positive moderate	Intensity of magnetization 'M' is positive large
8. χ is independent of temperature	$\chi = \frac{C}{T}$ curie law	$\chi = \frac{C}{T - T_c}$ curie Weiss law	$\chi = \frac{C}{T + T_N}$ curie law	$\chi = \frac{C}{T \pm T_c}$
9. 				
10. Ex: water, mercury, H_2 , Bi & organic materials	Ex: Cr, alkali metals, & transition metals	Ex: Transition & rare earth metals (Fe ,CO,Ni Gd) T_c -curie temperature	Ex: salts of transition elements (oxides of Ni ,Fe ,Cr) T_N - Neel temperature	Ex: Ferrites $(MnFeO_4)$

Curie temperature:

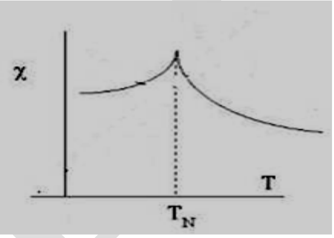
The temperature at which ferro magnetic material converts into para magnetic material is known as curie temperature.

Here T_c - Curie temperature.



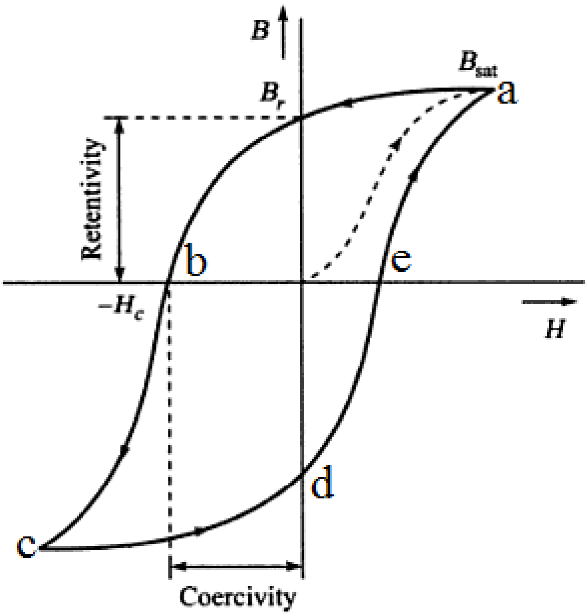
Neel temperature : The temperature at which the antiferro material susceptibility increases gradually with temperature and attains a maximum value is known as Neel temperature.

Here T_N - Neel temperature.



HYSTERESIS CURVE (Hysteresis curve for Ferro magnetic materials):

It means retardation. (or) Lagging of an effect behind the cause of the effect (or)the phenomenon of **B** lagging **H**. (or) **it** is also defined as a phenomenon of flux density (**B**)with the change in magnetic field strength (**H**) in a ferromagnetic material.



Explanation:

When the magnetic field H is applied on an unmagnetised ferromagnetic material the magnetic induction increases first rapidly and then slowly from **o** to **a**. The increase is nonlinear, after that the rate of induction slows down and attaining a saturation value **Bsat**, with further increase in H, there is no increase in B. If applied magnetic field H on the material is
decreased at this stage, naturally B decreases and B will not travel in that initial path, creates a new path.

When H is reduced to zero, B does not vanish, the value of B that remains in the material is called as **retentivity Br** or **reducemagnetisation or residual magnetization.**

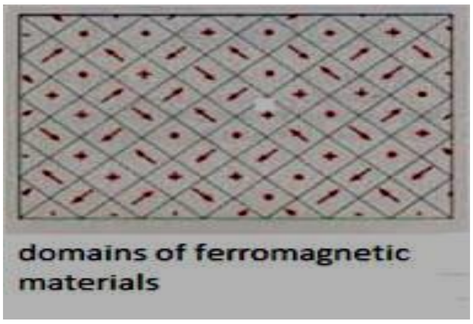
To remove retentivity in the material sufficient magnetic field Hc is applied in opposite direction then the retentivity Br becomes zero .This is (field) known as **coherceive field**.

On increasing H further B reaches saturation in the opposite direction denoted by ’c’. Then reducing H to zero B reaches‘d’ and then increasing H in the positive direction B reaches again to **Bsat** and a curve is obtained. This completes a closed loop called **hysteresis loop**.

It includes some area. This area indicates the amount of energy wasted in one cycle of operation. Further, though the magnetic field H is changed the curve repeats as **abcdea**.

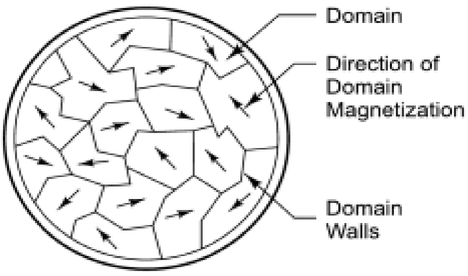
This hysteresis loop speaks about the characteristic nature of that ferromagnetic material.
Since a ferromagnetic is made up of domains, with the application of the magnetic field, the domains themselves rotate slowly in the field direction .In soft ferromagnetic materials; the area of the Hysteresis loop is a loss in the material called hysteresis loss. Hence, soft magnets are made with small area of hysteresis. In hard magnetic materials, this area is called the energy product which is large for permanent magnets.

Domain theory of Ferro magnetism:



In 1907, Weiss proposed domain theory to explain Ferro magnetism. According to this theory a single crystal of Ferro magnetic solid expresses a large number of small regions, and each region is spontaneously magnetized to saturation extent called a domain.

The domain size may vary from 10^{-6} to the entire volume of the crystal. In each domain spontaneous magnetization is due to parallel alignment of all magnetic dipoles and the direction of spontaneous varies from domain to domain. The resultant magnetization may hence be zero (or) nearly zero.

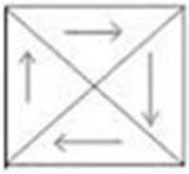


The spin magnetic moments of all atoms in a domain are oriented in a particular direction. However these regions themselves are oriented with respect to each other randomly, so that the total magnetization of the An magnetized sample contains domains. A Ferro magnetic material magnetizes when an external magnetic field is applied .This individual domains contribute total magnetization M of the specimen.

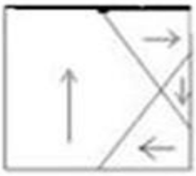
Becker suggested two independent processes by which magnetization of the specimen takes place,they are:

(i) <u>by rotations of domain walls:</u> The domains that are parallel (or) nearly parallel to the direction of applied magnetic field will grow in size at the cost of other domain.

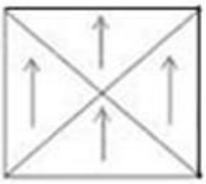
(ii) <u>by rotation of domains:</u> The magnetic moment of domains can rotate in the direction of applied field.



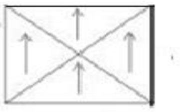
Domain orientation in the absence of Magnetic field



Due to weak Magnetic field

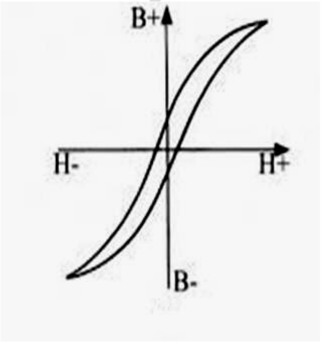
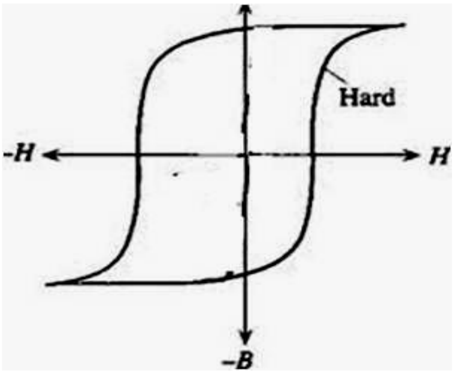


Due to strong applied field



Due to saturation of very high fields

Soft and Hard magnetic materials:

Soft magnetic materials	Hard magnetic materials
1.The materials which are easily magnetized or demagnetized are said to be soft magnetic material  	The materials which are very difficult to magnetize and demagnetize 
2. The nature of hysteresis loop of a soft	The nature of hysteresis loop is very broad and

material is very steep and have small loop area	have large loop area
3. Hysteresis loop is small hence lose is also small.	Since the area of curve is large ,the hysteresis loss is also large
4. Susceptibility and permeability values are very large.	Susceptibility and permeability values are very small
5. The coercivity and retentivity values are small	The coercivity and retentivity values are very high
6. These materials are free from irregularities or impurities	These materials have large amount of impurities and lattice defects
7. Iron – silicon alloys; Ni -Fe alloys Fe - CO alloys are example	Carbon steels, Tungsten steels, Chromium steels, CuNiCo, CuNiFe, AlNiCo alloys etc.
8. Uses: These are used in transformers, high frequency rotating machines	Uses: These are used as magnets for toys, compass needles, & permanent magnets
9. Resistivity of these materials is very high.	Resistivity of these materials is very low

UNIT V LASERS

Introduction:LASER stands for Light Amplification by Stimulated Emission of Radiation. Laser technology started with Albert Einstein in 1917, he has given theoretical basis for the development of Laser. The technology further evolved in 1960 when the very first laser called Ruby Laser was built at Hughes Research Laboratories by T.H. Mainmann.

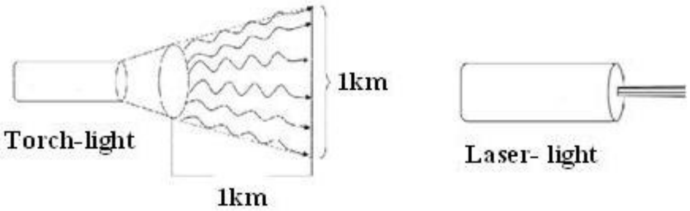
Characteristic of laser radiation:

The laser beam has the properties given below which distinguish it from an ordinary beam of light. Those are

1. Highly directional
2. Highly monochromatic
3. Highly intense
4. Highly coherence

1. Highly directional:

A conventional light source emits light in all directions. On the other hand, Laser emits light only in one direction. The width of Laser beam is extremely narrow and hence a laser beam can travel to long distances without spreading.



The directionality of laser beam is expressed in terms of divergence

$$\Delta\theta = \frac{r_2 - r_1}{d_2 - d_1}$$

Where r_1 and r_2 are the radii of laser beam spots at distances of d_1 and d_2 respectively from laser source.

2. Highly monochromatic:

A monochromatic source is a single frequency or single wavelength source of light. The laser light is more monochromatic than that of a convectional light source. This may be due to the stimulated characteristic of laser light. The band width of convectional monochromatic light source is 1000A^0. But the band width of ordinary light source is 10 A^0. For high sensitive laser source is 10^{-8} A^0.

3. Highly intense:

Laser light is highly intense than the conventional light. A one milli-Watt He-Ne laser is highly intense than the sun intensity. This is because of coherence and directionality of laser. Suppose when two photons each of amplitude ‘A’ are in phase with other, then young’s principle of superposition, the resultant amplitude of two photons is 2A and the intensity is 4a^2. Since in laser many numbers of photons are in phase with each other, the amplitude of the resulting wave becomes ‘nA’ and hence the intensity of laser is proportional to n^{2}A^2. So 1mw He-Ne laser is highly intense than the sun.

4. Highly coherence

A predictable correlation of the amplitude and phase at any one point with other point is called coherence.In case of conventional light, the property of coherence exhibits between a source and its virtual source where as in case of laser the property coherence exists between any two sources of same phase.

There are two types of coherence

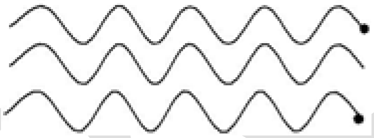
i) Temporal coherence
ii) Spatial coherence.

Temporal coherence (or longitudinal coherence):

The predictable correlation of amplitude and phase at one point on the wave train w .r. t another point on the same wave train, then the wave is said to be temporal coherence.

Spatial coherence (or transverse coherence):

The predictable correlation of amplitude and phase at one point on the wave train w. r .t another point on a second wave, then the waves are said to be spatial coherence (or transverse coherence).Two waves are said to be coherent when the waves must have same phase & amplitude.

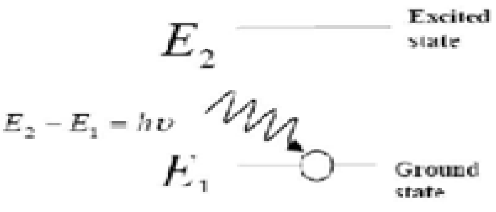


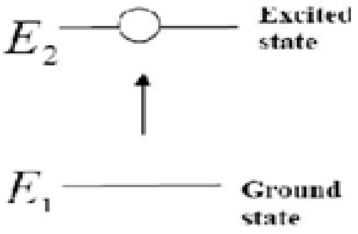
Interaction of light with matter and the three quantum processes:

When the radiation interacts with matter, results in the following three important phenomena. They are

(i)Induced or Stimulated Absorption
(ii)Spontaneous Emission
(iii)Stimulated Emission

Stimulated absorption (or) induced absorption (or) absorption: An atom in the lower energy level or ground state energy level (E_1) absorbs the incident photon and goes to excited state (E_2) as shown in figure below. This process is called induced or stimulated absorption.





Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if a photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation form ground state E_1 to excited state E_2 . This process is called stimulated absorption.

Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density of photons.

Spontaneous emission: The atom in the excited state returns to ground state emitting a photon of energy (E) = $E_2 - E_1 = h\nu$, without applying an external energy spontaneously is known as spontaneous emission.

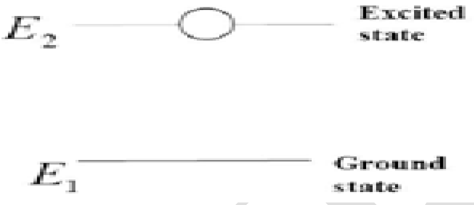
Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation form ground state E_1 to excited state E_2

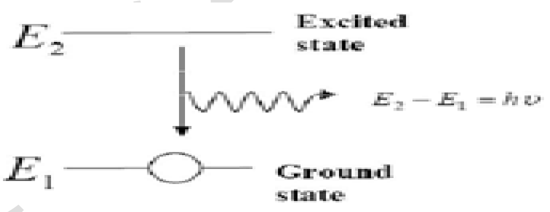
The excited atom does not stay for a long time in the excited state. The excited atom gets de-excitation after its life time by emitting a photon of energy $E_2 - E_1 = h\nu$. This process is called spontaneous emission.

The spontaneous emission rate depends up on the number of atoms present in the excited state.

The probability of spontaneous emission (P_{21}) is independent of $u(\vartheta)$.

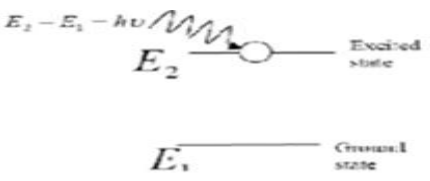
$$P_{21} = A_{21}$$

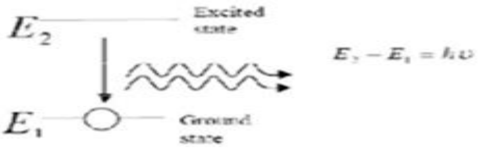




Stimulated-emission: The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission.

Stimulated emission was postulated by Einstein. Let E_1 and E_2 be the energies of ground and excited states of an atom. Let a Photon of energy E_2-E_1=hυ interacts with the excited atom with in their life time The atom gets de-excitation to ground state by emitting of another photon. These photons have same phase and it follows coherence. This phenomenon is called stimulated emission





Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of photons.

Comparison between Spontaneous and Stimulated emission:

Spontaneous emission	Stimulated emission
1. The spontaneous emission Was Postulated by Bohr.	1. The stimulated emission was Postulated by Einstein.
2. Additional photons are not required in spontaneous emission.	2. Additional photons are required in Stimulated emission.
3. One Photon is emitted in spontaneous emission.	3. Two photons are emitted in stimulated emission.
4. The emitted radiation is incoherent.	5. The emitted radiation is coherent.
5. The emitted radiation is less intense.	6. The emitted radiation is high intense.

Light Amplification:

Light amplification requires stimulated emission exclusively. In practice, absorption and spontaneous emission always occur together with stimulated emission. The laser operation is achieved when stimulated emission exceeds the other two processes due to its higher transitions rates of atomic energy levels.

Einstein coefficients & their relations:

Einstein coefficients:

Einstein mathematically expressed the statistical nature of the three possible radiative transition routes (spontaneous emission, stimulated emission, and absorption) with the so-called Einstein coefficients and quantified the relations between the three processes.

Let N_1 be the number of atoms per unit volume with energy E_1 and $\mathbf{N}_2$ be the number of atoms per unit volume with energy E_2. Let ‘n’ be the number of photons per unit volume at frequency ‘υ’ such that $E_2 - E_1 = h\vartheta$ Then, the energy density of photons $E = h\vartheta$.

- **Stimulated absoption**: When the photons interact with the atoms it leads to absorption transition which is called as stimulated absorption. Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density photons.

$$\text{Stimulated absorption rate} \propto N_1$$

$$\propto u(\vartheta)$$

$$= B_{12} N_1 \; u(\vartheta)$$

Where $\mathbf{B_{12}}$ is the Einstein coefficient of stimulated absorption.

- **Spontaneous emission**: The atom in the excited state returns to ground state emitting a photon of energy (E) = $E_2 - E_1$ =hν, without applying an external energy spontaneously is known as spontaneous emission. The spontaneous emission rate depends up on the number of atoms present in the excited state.

$$\text{Spontaneous emission rate} \propto N_2$$
$$= A_{21} N_2$$

Where A21 is the Einstein coefficient of spontaneous emission.

- **Stimulated emission**: The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission. Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of incident photons.

$$\text{Stimulated emission rate} \propto N_2$$
$$\propto u(\vartheta)$$
$$= B_{21} N_2 u(\vartheta)$$

Where $\mathbf{B_{21}}$ is the Einstein coefficient of stimulated emission.

If the system is in equilibrium, the rates of absorption transitions (Stimulated absorption) are equal to emission transitions(The rate of Spontaneous + Stimulated emission).
The rate of Absorption = The rate of Spontaneous + Stimulated emission

$$B_{12}N_1u(\vartheta) = A_{21}N_2 + B_{21}N_2u(\vartheta)$$

$$B_{12}N_1u(\vartheta) - B_{12}N_2u(\vartheta) = A_{21}N_2$$

$$u(\vartheta)(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$u(\vartheta) = \frac{A_{21N_2}}{B_{12N_1 - B_{21}N_2}}$$

$$u(\vartheta) = (\frac{A_{21}}{B_{21}} \times \frac{1}{\left[\frac{N_1}{N_2} \times \frac{B_{12}}{B_{21}} - 1\right]})$$

According to Boltzmann distribution law

$$N = N_0 e^{-\frac{E}{k_B T}}$$

$$N_1 = N_0 e^{-\frac{E_1}{k_B T}}$$

$$N_2 = N_0 e^{-\frac{E_2}{k_B T}}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\frac{E_1}{k_B T}}}{N_0 e^{-\frac{E_2}{k_B T}}} = e^{\frac{E_2 - E_1}{k_B T}} = e^{\frac{h\vartheta}{k_B T}}$$

$$u(\vartheta) = \frac{A_{21}}{B_{21}} \times \frac{1}{[e^{\frac{h\vartheta}{k_B T}}\left(\frac{B_{12}}{B_{21}}\right) - 1]} \text{-----}(1)$$

According to Planck's radiation formula the energy density of photon is given by,

$$u(\vartheta) = \frac{8\pi h\vartheta^3}{c^3} \times \left(\frac{1}{\left[e^{\frac{h\vartheta}{k_BT}} - 1\right]}\right) \text{-----}(2)$$

Comparing eq.(1) & eq.(2), we get

$$(i) \qquad \frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{c^3} \quad or \quad \frac{A_{21}}{B_{21}} \propto \vartheta^3$$

$$(ii) \qquad \frac{B_{12}}{B_{21}} = 1 \quad or, \quad B_{12} = B_{21}$$

Population inversion:

The number of atoms present in the excited (or higher) state is greater than the number of atoms present in the ground energy state (or lower state) is called population inversion.

Let us consider two level energy systems of energies E_1 and E_2 as shown in figure. Let N_1 and N_2 be the population (means number of atoms per unit volume) of E_1 and E_2 respectively.

According to Boltzmann's distribution the population of an energy level E, at temperature T is given by

$$N_i = N_o e^{\left(-\frac{E_i}{k_BT}\right)} \qquad where\ i = 1,2,3, \ldots . N_i$$

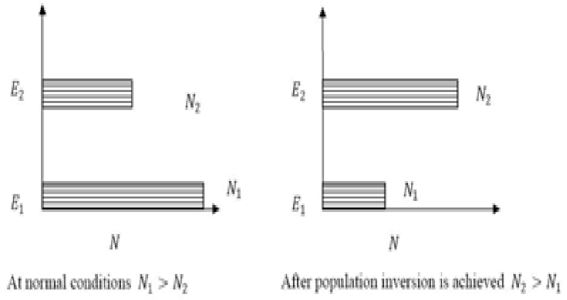
where 'N_0' is the number of atoms in ground or lower energy states & k is the Boltzmann constant.

From the above equation the population of energy levels E_1 & E_2 are given by

$$N_1 = N_o e^{\left(-\frac{E_1}{k_BT}\right)}$$

$$N_2 = N_o e^{\left(-\frac{E_2}{k_BT}\right)}$$

At ordinary conditions $N_1 > N_2$ i.e., the population in the ground or lower state is always greater than the population in the excited or higher states. The stage of making, population of higher energy level greater than the population of lower energy level is called population inversion i.e., $N_2 > N_1$.



At normal conditions $N_1 > N_2$ After population inversion is achieved $N_2 > N_1$

Meta stable state:

In general the number of excited particles in a system is smaller than the non excited particles. The time during which a particle can exist in the ground state is unlimited. On the other hand, the particle can remain in the excited state for a limited time known as life time. The life time of the excited hydrogen atom is of the order of 10^{-8}sec. However there exist such excited states in which the life time is greater than 10^{-8}sec. These states are called as Meta stable states.

Pumping mechanisms (or techniques):

Pumping:

The process of rising more no of atoms to the excited state by artificial means is called pumping.

A system in which population inversion is achieved is called as an active system. The method of raising the particles from lower energy state to higher energy state is called pumping. (or the process of achieving of population inversion is called pumping). This can be done by number of ways. The most commonly used pumping methods are

- Optical pumping
- Electrical discharge pumping
- Chemical pumping
- Injection current pumping

Optical pumping:

Optical pumping is used in solid laser. Xenon flash tubes are used for optical pumping. Since these materials have very broad band absorption, sufficient amount of energy is absorbed from the emission band of flash lamp and population inversion is created. Examples of optically pumped lasers are ruby, Nd: YAG Laser (Y_3 AL_5G_{12}) (Neodymium: Yttrium Aluminum Garnet), Nd: Glass Laser

Electrical discharge pumping:

Electrical discharge pumping is used in gas lasers. Since gas lasers have very narrow absorption band pumping them any flash lamp is not possible. Examples of Electrical discharge pumped lasers are He-Ne laser, CO_2 laser, argon-ion laser, etc

Chemical pumping:

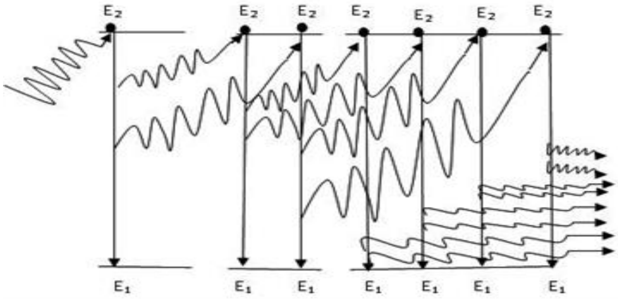
Chemical reaction may also result in excitation and hence creation of population inversion in few systems. Examples of such systems are HF and DF lasers.

Injection current pumping:

In semiconductors, injection of current through the junction results in creates of population inversion among the minority charge carriers. Examples of such systems are InP and GaAs.

Principle of laser/lasing action:

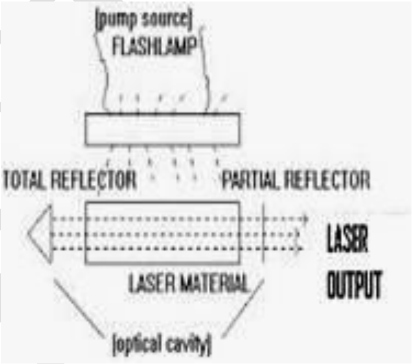
Let us consider many no of atoms in the excited state. Now the stimulating photon interacts with any one of the atoms in the excited state, the stimulated emission will occur. It emits two photons, having same energy & same frequency move in the same direction. These two photons will interact with another two atoms in excited state & emit 8-photons. In a similar way chain reaction is produced this phenomenon is called “Principle of lasing –action”. We get a monochromatic, coherent, directional & intense beam is obtained. This is called laser beam. This is the principle of working of a laser.



Components of a LASER:

Any laser system consists of 3-important components. They are

(i) Source of energy or pumping source
(ii) Active-medium (Laser Material)
(iii)Optical cavity or resonator



(i) Energy Source : It supply energies & pumps the atoms or molecules in the active medium to excited states. As a result we get population inversion in the active medium which emits laser.
Ex: Xenon flash lamp, electric field.

(ii) Active medium: The medium in which the population inversion takes place is called as active medium.
Active-centre: The material in which the atoms are raised to excited state to achieve population inversion is called as active center.

(iii) Optical-cavity or resonator: The active medium is enclosed between a fully reflected mirror & a partially reflective mirror. This arrangement is called as cavity or resonator. As a result, we get highly intense monochromatic, coherence laser light through the non-reflecting portion of the mirror.

DIFFERENT TYPES OF LASERS

On the basis of active medium used in the laser systems, lasers are classified into several types

I. Solid lasers : Ruby laser, Nd;YAG laser, Nd;Glass
II. Liquid lasers : Europium Chelate laser, $SeOCl_2$
III. Gas lasers : CO_2, He-Ne, Argon-Ion Laser
IV. Dye lasers : Rhodamine 6G
V. Semiconductor lasers : InP, GaAs.

RUBY LASER :

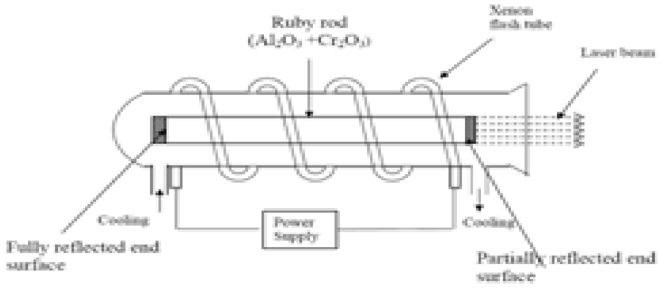
Ruby laser is a three level solid state laser and was developed by Mainmann in 1960. Ruby (Al_2O_3+Cr_2O_3) is a crystal of Aluminium oxide, in which 0.05% of Al^{+3} ions are replaced by the Cr^{+3} ions. The colour of the ruby rod is pink. The active medium in the ruby rod is Cr^{+3} ions.

Principle or Characteristics of a ruby laser:

Due to optical pumping, the chromium atoms are raised to excited states then the atoms come to metastable state by non-radiative transition. Due to stimulated emission the transition of atoms takes place from metastable state to ground state and gives a laser beam.

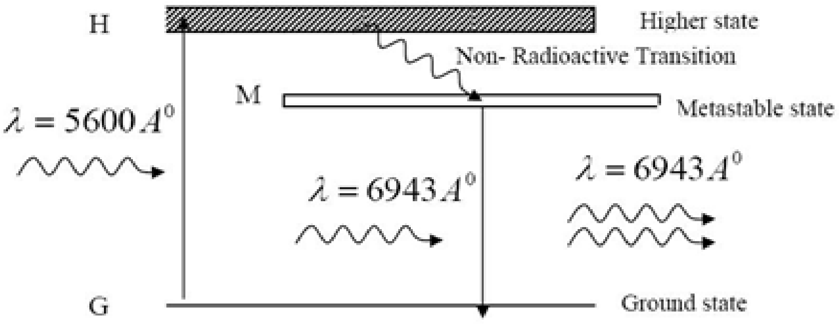
Construction:

- In ruby laser 4cm length and 5mm diameter rod is generally used.
- Both the ends of the rods are highly polished and made strictly parallel.
- The ends are silvered in such a way, one becomes partially reflected and the other end fully reflected.
- The ruby rod is surrounded by xenon flash tube, which provides the pumping light to excite the chromium ions in to upper energy levels.



- Xenon flash tube emits thousands joules of energy in few milli seconds, but only a part of that energy is utilized by the chromium ions while the rest energy heats up the apparatus.
- A cooling arrangement is provided to keep the experimental set up at normal temperatures.

Working:



- The energy level diagram of chromium ions is shown in figure.
- The chromium ions get excitation into higher energy levels by absorbing of $5600A^0$ of wave length radiation.

- The excited chromium ions stay in the level H for short interval of time (10^{-8} Sec).
- After their life time most of the chromium ions are de-excited from H to G and a few chromium ions are de-excited from H to M.
- The transition between H and M is non-radioactive transition i.e. the chromium ions gives their energy to the lattice in the form of heat.
- In the Meta stable state the life time of chromium ions is 10^{-3} sec.
- Due to the continuous working of flash lamp, the chromium ions are excited to higher state H and returned to M level.
- After few milli seconds the level M is more populated than the level G and hence the desired population inversion is achieved.
- The state of population inversion is not a stable one.
- The process of spontaneous transition is very high.
- When the excited chromium ion passes spontaneously from H to M it emits one photon of wave length $6943A^0$.
- The photon reflects back and forth by the silver ends and until it stimulates an excited chromium ion in M state and it to emit fresh photon in phase with the earlier photon.
- The process is repeated again and again until the laser beam intensity is reached to a sufficient value.
- When the photon beam becomes sufficient intense, it emerges through the partially silvered end of the rod.
- The wave length $6943A^0$ is in the red region of the visible spectrum on returning to ground state (G).

Uses of Ruby laser:

- Used in distance measurement using ‘pulse echo technique’
- Used for measurement of plasma properties such as electron density and temperature.
- Used to remove the melanin of the skin.
- Used for recording pulsed holograms.
- Used as target designators and range finders in military.

Draw backs of Ruby laser:

- It requires high pumping power.
- The efficiency of ruby laser is very small. It is a pulse laser.

He-Ne LASER:

It was discovered by A. Javan & his co-workers in 1960. It is a continuous wave gas laser. It consists of mixture of He & Ne in 10:1 ratio as a active medium.

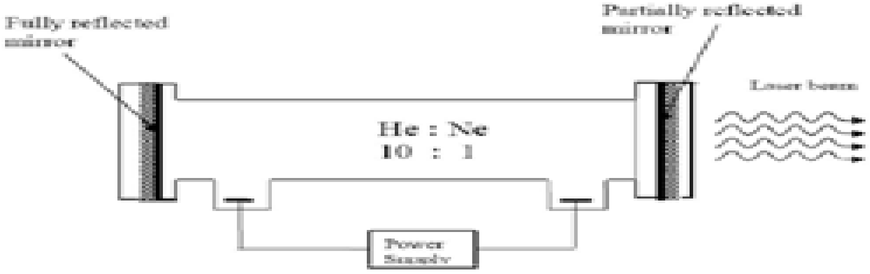
Principle/Characteristics of He-Ne laser:

This laser is based on the principle of stimulated emission, produced in the He & Ne. The population inversion is achieved due to the interaction between He & Ne gases. Using gas lasers, we can achieve highly coherent, directional and high monochromatic beam.

Construction:

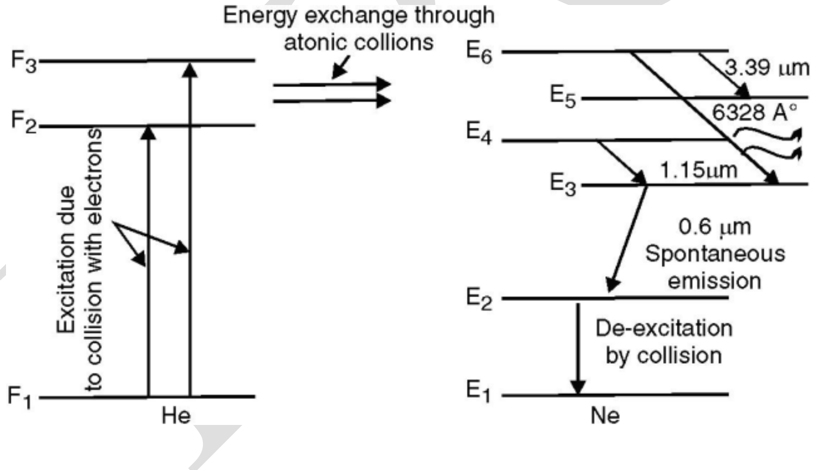
- In He-Ne gas laser, the He and Ne gases are taken in the ratio 10:1 in the discharge tube.

- Two reflecting mirrors are fixed on either ends of the discharge tube, in that, one is partially reflecting and the other is fully reflecting.
- In He-Ne laser 80cm length and 1cm diameter discharge tube is generally used.
- The output power of these lasers depends on the length of the discharge tube and pressure of the gas mixture.
- Energy source of laser is provided by an electrical discharge of around 1000V through an anode and cathode at each end of the glass tube.



Working:

- When the electric discharge is passing through the gas mixture, the electrons accelerated towards the positive electrode.
- During their passage, they collide with He atoms and excite them into higher levels.
- F_2 and F_3 form F_1. In higher levels F_2 and F_3, the life time of He atoms is more.
- So there is a maximum possibility of energy transfer between He and Ne atoms through atomic collisions.
- When He atoms present in the levels F_2 and F_3 collide with Ne atoms present ground state E_1, the Ne atoms gets excitation into higher levels E_4 andE_6.



- Due to the continuous excitation of Ne atoms, we can achieve the population inversion between the higher levels E_4 (E_6) and lower levels E_3 (E_5).
- The various transitions $E_6 \rightarrow E_5$, $E_4 \rightarrow E_3$ and $E_6 \rightarrow E_3$ leads to the emission of wavelengths $3.39A^0$, $1.15\ A^0$ and $6328A^0$.
- The first two corresponding to the infrared region while the last wavelength is corresponding to the visible region.
- The Ne atoms present in the E_3 level are de-excited into E_2 level, by spontaneously emission of photon.

- When a narrow discharge tube is used, the Ne atoms present in the level E_2 collide with the walls of the tube and get de-excited to ground level E_1.

Uses of He-Ne laser:

- Used in laboratories foe all interferometric experiments.
- Used widely in metrology in surveying, alignment etc.
- Used to read barcodes and He-Ne laser scanners also used for optical character recognition.
- Used in holography.

Applications of lasers:

Due to high intensity, high mono-chromaticity and high directionality of lasers, they are widely used in various fields like

1. communication
2. computers
3. chemistry
4. photography
5. industry
6. medicine
7. military
8. scientific research

1. Communication:

- In case of optical communication, semiconductors laser diodes are used as optical sources.
- More channels can be sent simultaneously Signal cannot be tapped as the band width is large, more data can be sent.
- A laser is highly directional and less divergence, hence it has greater potential use in space crafts and submarines.

2. Computers :

- In LAN (local area network), data can be transferred from memory storage of one computer to other computer using laser for short time.
- Lasers are used in CD-ROMS during recording and reading the data.

3. Chemistry :

- Lasers are used in molecular structure identification.
- Lasers are also used to accelerate some chemical reactions.
- Using lasers, new chemical compound can be created by breaking bonds between atoms or molecules.

4. Photography :

- Lasers can be used to get 3-D lens less photography.
- Lasers are also used in the construction of holograms.

5. Industry :

- Lasers can be used to blast holes in diamonds and hard steel.
- Lasers are also used as a source of intense heat.

- Carbon dioxide laser is used for cutting drilling of metals and nonmetals, such as ceramics plastics, glass etc.
- High power lasers are used to weld or melt any material.
- Lasers are also used to cut teeth in saws and test the quality of fabric.

6. Medicine :

- Pulsed neodymium laser is employed in the treatment of liver cancer.
- Argon and carbon dioxide lasers are used in the treat men of liver and lungs.
- Lasers used in the treatment of Glaucoma.

7. Military :

- Lasers can be used as a war weapon.
- High energy lasers are used to destroy the enemy air-crafts and missiles.
- Lasers can be used in the detection and ranging likes RADAR.

8. Scientific research:

- Lasers are used in the field of 3D-photography.
- Lasers used in Recording and reconstruction of hologram.
- Lasers are employed to create plasma.
- Lasers used to produce certain chemical reactions.
- Lasers are used in Raman spectroscopy to identify the structure of the molecule.
- Lasers are used in the Michelson- Morley experiment.
- A laser beam is used to confirm Doppler shifts in frequency for moving objects.